Endogenous Growth and Excess Variety

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Abstract

A large family of endogenous growth models reconciles rising populations with constant growth rates by positing that, as population grows, research effort is spread over an ever wider range of goods. In such models, if the range of goods were fixed, growth would accelerate. In fact, under standard assumptions, shrinking the range of goods could instantaneously yield infinite output. This result is avoided only by positing severely diminishing returns to research inputs. This has implications for the plausibility of endogenous growth theory in its current form, as well as for the growth impact of automation.

1 Introduction

Two schools in modern growth theory — The common understanding since Romer (1990) is that output per person is governed in the long run by the quantity of technology, and that technology grows due to investments in R&D. In Romer's and other early "endogenous" growth models (Grossman and Helpman (1991), Aghion and Howitt (1992)), the growth rate is constant if the population is constant but increases in the population size, fixing the share of people engaged in research. As Jones (1995) observes, however, populations have grown greatly over the last century, and the population of researchers has grown even more quickly, yet the growth rate has not risen. Modern growth theory offers two primary approaches to reconciling these facts.

"Semi-endogenous" growth models (Jones (1995), Kortum (1997), Segerstrom (1998)) posit that as technology advances, further advances—i.e. further proportional productivity improvements—get harder to find. A constant growth rate therefore requires population growth. With a constant population, technology growth would eventually slow.

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"Second-wave endogenous" (I will write "SWE") growth models posit instead that technology grows exponentially with a constant or with a growing population. The idea, with minor variations, is that process efficiency—the quantity of a given good[1] producible with given labor and/or capital inputs—grows exponentially with constant research effort, as in a first-wave endogenous model; but when population grows, we develop more goods, leaving research effort per good fixed. Improvements in process efficiency are called "vertical innovations" and increases in good variety are called "horizontal innovations". Variety may or may not be desirable, so an increase to the population size may or may not yield a level effect, but in either case it does not yield a growth effect. Likewise exponential population growth may raise the technology growth rate from the positive rate that obtains when the population is fixed, by adding a horizontal dimension to the constant vertical dimension, but even if so, the result is still a constant rather than a rising growth rate.

Young (1998) introduced the SWE approach, roughly contemporaneously with Peretto (1998), Dinopoulos and Thompson (1998), and Aghion and Howitt (1998, ch. 12.2). It continues to be developed. Aghion, Bergeaud, Boppart and Brouillette (2025) offer one recent iteration, designed to accommodate evidence from Bloom et al. (2020) that constant process efficiency growth cannot be sustained with constant research inputs at the good level. Massari and Peretto (2025) offer another, designed to avoid the critique that variety responds to population precisely enough to maintain exponential growth only under knife-edge conditions.[2]

A challenge for the SWE approach — The literature to date has not discussed an important challenge to the SWE approach. If the growth rate increases in research effort per good, then since fixing the range of goods is feasible, population growth makes an ever-rising growth rate feasible. Shrinking the range of goods allows for even faster growth, even with a fixed population. In fact, almost every implementation of the approach yields an "extreme inefficiency result": that though the equilibrium growth rate is constant, as historically observed, **it is feasible to generate infinite utility and/or infinite consumption in arbitrarily little time**.

This point seems to have gone unremarked in part because Proposition 5 of Dinopoulos and Thompson (1998) claims that an optimal growth path exists and can be implemented with an appropriate R&D subsidy; footnote 6 of Aghion and Howitt (1998, ch. 12.2) claims that the "counterintuitive welfare result" that "the optimal number of different products is vanishingly small" can be eliminated with a proposed tweak to the model; and one or the other of these models closely, for our purposes, resembles all subsequent SWE literature. As we will see, if the product range can shrink, the first claim is false and the second is true only given severely diminishing returns to research inputs. Neverthe-

[1] Or its "quality", which is modeled as equivalent.

[2] Not every modern endogenous growth model is SWE. For instance, Dinopoulos and Syropoulos (2007) and Dinopoulos et al. (2023) study the possibility that larger populations do not yield faster growth rates because the increase in research capacity is offset by an increase in rent-seeking behavior.

less, they seem to have closed interest in the efficiency question. No subsequent SWE paper attempts to solve for an optimal growth path.[3,4]

The extreme inefficiency result can be avoided by positing that vertical innovation faces severely diminishing returns to research labor, so that in effect, a larger population can only be employed productively in research if the range of goods widens. In particular, to render double-exponential growth infeasible, the function from investments in process efficiency to the rate of vertical innovation must be sublogarithmic. Young (1998) takes this approach. In principle, the result can also more bluntly be avoided by positing that the maximum feasible rate of vertical innovation actually *decreases* in population per good. Aghion et al. (2025) make a modeling choice that sometimes has this consequence, on inspection, though it is difficult to see how this implication could be justified.

Outline — In Section 2 I cover "research labor" models, i.e. those in which labor is the direct input to vertical innovation. In Section 3 I cover "lab equipment" models, i.e. those in which output is the direct input. I believe that the sections encompass every SWE model with a distinctive feature in any way relevant to the inefficiency results.

Throughout, by reframing the models and drawing out their similarities, I hope to illustrate that the tendency for SWE models to exhibit extreme inefficiency is not a mathematical curiosity but follows directly from the central premise of the approach. If growing variety is indeed defusing explosive growth, then if we are not careful we are liable to conclude that slowing or shrinking variety can generate it, and the faster the better. This conclusion is avoidable, but only by making a modeling choice with its own arguably undesirable features.

In Section 4, I evaluate the plausibility of the two conclusions compatible with SWE growth theory: (i) that explosive growth is technologically feasible but goes unrealized due to market failure and (ii) that the returns to innovation diminish more quickly than is usually appreciated. I then note that these possibilities respectively widen the range of answers to how growth will change if production and R&D are much more fully automated, as many technologists forecast they soon will be (Grace et al., 2024). Conclusion (i) implies that the prospect of building agents that can coordinate well enough to avoid the market failure in question—even if they are relatively few and unproductive—constitutes a new channel through which automation could yield explosive growth. Conclusion (ii) implies that even full automation may have relatively contained economic impact.

[3] Aghion et al. (2025) do solve for optimal policy under constraints. Peretto (1999) studies a modification to Peretto (1998) in which there is a finite set of firms each period rather than a monopolistically competitive continuum; he then considers the optimal number of firms when R&D per firm is profit-maximizing, but not efficiency more generally.

[4] The point has certainly not gone unremarked simply because it (and perhaps some workaround to it) is too well-known. None of the professors or graduate students at a recent meeting of the Stanford growth seminar had been aware of this point, and it is at the suggestion of one of them that I have written this. The only source of which I am aware that makes a similar point is Davidson (2021), who points out that growth can be hyperbolic in the model of Peretto (2018) with a fixed good range under a parameter restriction.

2 Research labor models

2.1 Without severely diminishing returns

This section discusses inefficiency in Dinopoulos and Thompson (1998), Peretto (1998), Peretto and Smulders (2002), and Peretto and Connolly (2007). We will start with a simplification of Peretto (1998) and discuss its implications in depth, because this model is in many ways the clearest and simplest, and the one after which subsequent literature is most closely patterned. We will then observe that the modifications to this model that make each paper distinctive do not change these implications.

Benchmark: Peretto (1998) (simplified) — A representative individual has log Dixit-Stiglitz preferences:

$$U = \int_0^\infty e^{(\gamma-\rho)t} \ln c_t \, dt, \;\; \rho > \gamma \geq 0; \tag{1}$$

$$c_t = \left(\int_0^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \;\; \varepsilon > 1, \tag{2}$$

where $c_{it} \geq 0$ denotes the consumption of good i at t, $N_t > 0$ the range of goods available at t, $\rho > 0$ the discount rate, and $\gamma \in [0, \rho)$ the population growth rate.

Production per person of good i is

$$c_{it} = A_{it} L_{it} / L_t, \tag{3}$$

where $L_i \geq 0$ denotes the labor producing good i and $A_i \geq 0$ denotes its productivity.[5]

Let $s \in (0,1)$ denote the "research share", i.e. the share of the population engaged in research, so that sL_t is the quantity of research labor at t and

$$\int_0^{N_t} L_{it} di = (1-s) L_t$$

is the quantity of production labor. If A_{it} equals a constant value A_t for all $i \in [0, N_t]$, then it is the efficient and the equilibrium outcome for production labor and consumption to be distributed evenly across goods. Fixing s, the consumption aggregate then reduces to

$$c_t \propto A_t N_t^\sigma, \quad \sigma \equiv \frac{1}{\varepsilon - 1} > 0. \tag{4}$$

Assume for simplicity that this symmetry holds at time 0.

Process efficiency grows according to

$$\dot{A}_{it} \propto A_{it} S_{it}^\lambda, \;\; \lambda > 0, \tag{5}$$

[5] This can be found by rearranging Peretto's equation 5 and letting $A_i \equiv Z_i^\theta$.

where $S_i \geq 0$ denotes the quantity of labor doing process research on good i. Since this is the only kind of research we will need to consider, $\int_0^N S_i di = sL$.

Result 1: Infinite utility via double-exponential growth

If we fix N and s, so that S_{it} grows at rate γ for all i, then $\dot{A}_{it}/A_{it}$ itself grows at rate $\gamma\lambda$ for all i, and A_t does as well. More generally, if we fix the fraction of the population engaged in research and allocate it evenly across the product range, symmetry gives us

$$\dot{A}_t \propto A_t(L_t/N_t)^\lambda, \tag{6}$$

so that if we enforce a constant $g_N \leq \gamma$, we can sustain a path of $g_{A,t}$ that grows exponentially at rate $\lambda(\gamma - g_N)$. The consumption aggregate c thus grows asymptotically at this double-exponential rate; i.e. $\ln c$ asymptotically grows at rate $\lambda(\gamma - g_N)$. By (1), if

$$\begin{aligned} g_N &< \gamma - \frac{\rho - \gamma}{\lambda} \\ \implies \lambda(\gamma - g_N) &> \rho - \gamma, \end{aligned}$$

the payoff is infinite.

As a corollary, if

$$\rho < \gamma(1 + \lambda), \tag{7}$$

the infinite payoff is achievable even without shrinking N.

Interestingly, condition (7) lies just on the edge of the conventional parameter values $\lambda = 1$, $\rho \approx 0.02$, and $\gamma \approx 0.01$. The conclusion that infinite utility is achievable merely by banning new goods and sustaining research effort on existing goods, in a Peretto (1998)-style model, is therefore sensitive to the calibration. Nevertheless, because utility is continuous in the parameters, the weaker conclusion that banning new goods would be immensely desirable is relatively robust.[6]

Note that we have not accounted for the fact that foregoing horizontal innovation saves entry costs (and have certainly not assumed that shrinking the range of goods would generate negative entry costs). We will ignore saved entry costs through the rest of the paper as well.

Result 2: Infinite output in no time

The connection between double-exponential growth and what I call "extreme inefficiency" relies on the choice of log utility. If we instead use a utility function featuring more steeply diminishing returns, it is less obvious that exponential growth given (4)–(5)

[6]The conclusion that immense utility would have been achieved by halting new product development historically, keeping all research effort devoted to vertical innovation, is greatly strengthened if we accept the estimate from Bloom et al. (2020) that $s_t L_t$ has grown not by 1% but by over 4% for almost a century.

is inefficient at all. A constant, high growth rate may be preferable to one that begins low but grows exponentially.

If we allow N to shrink, however, (4) and (6) imply that for any $T > 0$, there is a feasible growth path with

$$\lim_{t \to T^-} c_t = \infty. \tag{8}$$

For example, suppose the product range shrinks so that

$$N_t \propto (T-t)^{2/\lambda} \text{ for } t < T. \tag{9}$$

Since T may be arbitrarily small, assume for simplicity that L is fixed. Substituting $1/N_t$ for S_{it} into (5)—the "$\propto$" allows this if we fix NS_i—and recalling that we will have $A_{it} = A_t$ for all $i \leq N_t$, we have

$$\dot{A}_t \propto A_t(T-t)^{-2} \text{ for } t < T.$$

Solving this differential equation for A yields

$$A_t \propto e^{k\frac{t}{T-t}}, \; k > 0 \text{ for } t < T. \tag{10}$$

Substituting (9) and (10) into (4) yields (8).

The model implies not only that infinite output is feasible in finite time but that it is feasible in *arbitrarily little* time, and has been for as long as the model has resembled reality. This is surely implausible, but strictly speaking it does not prove that exponential growth is extremely inefficient for any increasing utility function. This is because, near time 0, sufficiently rapid decreases to N cut consumption more quickly than the not-yet-rapid increases to A raise it, so that c falls temporarily before rising boundlessly near T (and falls nearer to zero the lower T is). Extreme inefficiency would however follow immediately if the model were extended in either of two ways. First, we could allow consumption to be saved and consumed later in even the most marginal way, e.g. by introducing an unproductive form of capital with an arbitrarily high depreciation rate. Second, we could stipulate that the economy can partition itself and instruct one part to carry out the brief research program described above while the other part subsidizes it.

What if variety is lower-bounded?

One might reasonably object that it is infeasible to shrink the range of goods below some $\underline{N} > 0$. This might be because in practice goods are discrete,[7] or because some goods are necessary, such that the love of variety captured by a Dixit-Stiglitz aggregator only approximates our preferences once these basic needs are met.

However, SWE models are designed to explain the stylized fact of long-run exponential growth, famously documented by Kaldor (1961) as persisting since the 19th century, so we must take it as feasible to shrink variety at least to the level that obtained

[7]Or firms; see Peretto (1999) or the Salop circle model of Young (1995).

when the trend began. Use the conventional benchmarks $\gamma = 0.01$ and $\sigma = 1/3$.[8] Then population—and, on an SWE account, the "good range" as measured in the relevant sense—150 years ago equaled

$$N_{1875}/N_{2025} = e^{-0.01\cdot 150} \approx 22\%$$

their current values. If output per person has grown at rate $g_A + 0.01/3 = 0.02$, then $g_A \approx 0.017$ at the current ratio of population per good. Multiplying the good range by 0.22 would thus multiply output initially by $0.22^{1/3} \approx 0.6$ but, if $\lambda = 1$, multiply g_A by $1/0.22 \approx 4.55$. The growth rate would thus rise from 2% to approximately 7.7%. *Even if g_A stayed fixed*, so that population growth did not raise g_A further, output per person would return to its initial level within 6.7 years and quadruple its initial level within 25 years.

If we accept the estimate from Bloom et al. (2020) that $s_t L_t$ has grown not by 1%/year but by over 4%/year at least since the 1930s, the result is starker. Supposing that variety has grown at 4%/year since 1935, we have

$$N_{1935}/N_{2025} = e^{-0.043\cdot 90} \approx 2\%.$$

By the calculation above, in this case we have historically seen $g_A = 0.02 - 0.043/3 \approx 0.57\%$. Cutting variety by a factor of 50 would multiply output by 0.27 but multiply g_A by a factor of 50. Again, even if g_A remains fixed, output per person returns to its initial level within 2.3 years, and within 15 years it rises by a factor well over 1,000.

Even the first proposal would be highly desirable given log utility with $\rho = 0.02$. Remarkably, relative to constant growth at 2% per year, multiplying output by 0.6 and raising the growth rate to 7.7% per year increases the payoff (1) by as much as a permanent level effect of a factor of 167. Implementing such a radical shift to the growth path would surely be very costly in practice, but any frictions introduced to the model must be severe indeed to overcome this benefit. Furthermore, the rapid growth path would be significantly more desirable if we account for the possibility of consuming goods in positive quantities after vertical innovation on them has halted, or, as noted in the discussion of Result 2 above, if we can temporarily subsist on accumulated capital.

For the rest of the paper, for simplicity, we will not introduce a lower bound on variety. We will trust that when the inefficiency results apply to an SWE model as written, reasonable variety-bounding constraints on the model do not overturn the conclusion that historical growth has been drastically inefficient.

Peretto (1998) proper — The only relevant difference from the "benchmark" model is that, in place of (5), process efficiency grows according to

$$\dot{A}_{it} \propto A_{it}^{1-\psi} A_t^{\psi} S_{it}^{\lambda}, \;\; \psi \in (0,1), \;\; \lambda > 0. \tag{11}$$

[8] See e.g. Aghion et al. (2025).

A_t denotes a consumption-share-weighted, CES/CRS aggregate of A_{it} across $i = 0$ to N_t, so the relationship $A_{it} = A_t$ ($i \leq N_t$) is maintained on the growth paths proposed in Results 1 and 2, and (11) reduces to (5).[9]

It is worth emphasizing that if N shrinks, the range of goods shrinks *from above*. Thus if good i exists at time t, then good $j < i$ must as well, as it does on any growth path conventionally considered. That is, the conclusion that it is feasible to produce astronomical growth by discarding products does not involve violating an implicit assumption that simpler goods (or the knowledge needed to produce them efficiently) must be maintained to support the production of more complex goods, or anything of that kind. We are simply assuming that if N is small, A_i ($i \leq N$) grows as quickly as if N had always been that small. We are following the letter of the model, and it seems hard to argue that we are not following the spirit.

To elaborate on this point: because A is a *share-weighted* average of the $\{A_i\}$, prohibiting the production of goods $i > \underline{N}$ really does eliminate them in the sense relevant to (4)–(6). A modified model could posit that if a good has ever been developed, it can drag down the productivity of pre-existing goods even after it has been abandoned, but this would be altogether unmotivated. Peretto's justification for including A in the production function for A_i is that "[w]hen one firm generates a new idea to improve its own production process, it also generates general-purpose knowledge that other firms exploit in their own research efforts" (p. 287).

To be sure, if Peretto (1998) is to be well-motivated, the above motivation must be incomplete. Suppose (i) the range of goods rises from $\underline{N}$ to $\overline{N}$, fixing A, and then (ii) A_j doubles only for $j \leq \underline{N}$. Expression (5) implies that $\dot{A}_i$ ($i \leq \underline{N}$) rises by less than if (ii) had occurred alone, even though firm i has access to knowledge produced by firms $j > \underline{N}$. Peretto does not discuss this point; he justifies making $\dot{A}_i$ depend on *average* knowledge across firms (rather than a statistic always non-decreasing in N) only "so that a steady state with constant growth is feasible" (p. 287). However Aghion and Howitt (1998, pp. 407–8) do acknowledge that an SWE model in which one firm's technology has positive spillovers must also allow for negative spillovers in this way, to neutralize scale, and they offer two justifications for accepting this conclusion. First, a wider variety of goods "makes life more complicated". Second, it raises "thin-market transaction costs". The modeling result relevant here—that cutting N allows A_i ($i < N$) to grow as quickly as if N had been low all along—is fully compatible with both arguments.

Finally, Massari and Peretto (2025) introduce a model (discussed in Section 3) in which the exponents on A_i and A in the right-hand side of (11) may sum to less than 1, but exponential growth is sustained with a constant population because N_t shrinks exponentially, so that in symmetric equilibrium, average process efficiency A grows exponentially.

Peretto and Smulders (2002) — The only relevant difference from the benchmark is that in

[9] Also, incidentally, Peretto considers only the $\lambda = 1$ case.

place of (5) we have[10]

$$\dot{A}_{it} \propto A_{it}\left(\frac{N_t}{a+N_t}\right)^{\psi} S_{it}^{\lambda}, \ \psi \in (0,1), \ \lambda > 0 \ \text{ for } \ N_t \geq N_0 > -a.$$

The large parenthetical term may increase or decrease in N_t, depending on the sign of a (though it is asymptotically constant as $N_t \to \infty$). It is microfounded by a model in which firms have positive spillovers on each other's research, but the size of the spillovers depends on how "far apart" the firms are in a space of technology processes, but expansions to the number of firms may also, in a sense, so widen the spread among them that the aggregate spillovers received by any given firm decrease.

Because the magnitude of firm j's spillovers on i does not increase as j grows more technologically advanced (as this would generate scale effects), it does not decrease if we halt j's technological development altogether. We may trivially fix N itself but allocate consumption and research labor over the good range $[0, \underline{N}_t]$ with $\underline{N}_t \to 0$. The inefficiency results follow immediately.

Peretto and Connolly (2007) — The only relevant difference from the benchmark is that in place of (3) we have

$$c_{it} = A_{it}(L_{it} - f)/L_t,$$

where $f > 0$ denotes a fixed cost, in units of labor, that must be paid to produce each good in a given period. Because fewer goods means fewer fixed costs, this tweak only strengthens the inefficiency results.

Dinopoulos and Thompson (1998) — There are two relevant differences from Peretto (1998).

First, $\dot{A}_i$ grows in proportion not to A_i but to a simple (not share-weighted) average of technology levels across goods:

$$A \equiv \int_0^N A_i di \Big/ N. \tag{12}$$

The assumption that it is feasible to shrink the range of goods, as Result 2 and in some cases Result 1 requires, is therefore a substantive one which the paper leaves ambiguous. It depends on whether, if a firm chooses to stop producing good j and/or consumers choose to stop consuming it, j still "constitutes part of the product range", such that stagnation in A_j slows the research process for goods $i < j$ (though it would not have if good j had never been invented). Following the discussion above on the implications of shrinking N, I maintain that there is no coherent motivation—and no motivation proposed in the literature—for the idea that a nonexistent good generates negative spillovers. I will therefore argue that on the most natural reading of the model, shrinking N is feasible.

[10]As above, this can be found by letting $A_i \equiv Z_i^{\theta}$ from the paper's fifth equation on, and the authors consider only the $\lambda = 1$ case. Our a is the authors' $J_0/\delta - N_0$ from their equation 19.

Second, each A_i does not grow deterministically and continuously as in (5), but in jumps that arrive stochastically with frequency proportional to S_i^λ. Because the set of goods is a continuum, however, this stochasticity does not appear in the aggregate, and (6) is precisely maintained.[11]

2.2 With severely diminishing returns

This section discusses Young (1998), the only labor-based SWE model to which the inefficiency results do not apply.

The model (simplified) — The model is set in discrete time. The utility function is the discrete-time analog to (1), paired with the standard Dixit-Stiglitz aggregator (2)–(4). Process efficiency however is given by

$$A_{it} - A_{it-1} \propto A_{it-1} \ln(S_{it}/f) \;\text{ for }\; S_{it} \geq f, \tag{13}$$

where f is the fixed labor cost of producing the good. Note that if a good is produced at t all, its process efficiency does not fall.[12]

A continuous-time analog — We will begin by considering a continuous-time analog to the model above. This will allow us to identify how severely the returns to research labor must diminish in order for a research-labor-based SWE model to escape extreme inefficiency, in terms more clearly consistent with the rest of the literature.

Suppose

$$\dot{A}_{it} = mA_{it} \ln(S_{it}/f),\; m > 0. \tag{14}$$

Even the logarithmic research function does not always escape Result 1. The result is now less obvious: infinite output in finite time is now unachievable, and merely enforcing $g_N < \gamma$ now generates linear growth in g_A, so that $\ln c$ grows quadratically rather than exponentially. But if $m > \sigma(\rho - \gamma)$, an infinite payoff is achieved by enforcing $N_t \propto e^{-e^{nt}}$ for $n > \rho - \gamma$. We will work through this point because doing so will illustrate why, given a logarithmic research function, growth cannot be faster than double-exponential. This in turn will illustrate why, given log utility, an infinite payoff is infeasible for all parameter values only if the research function is *sub*logarithmic.

Letting A denote a (perhaps share-weighted) average of $\{A_i\}$, and fixing $S_i = 1/N$,[13]

$$c_t \propto A_t N_t^\sigma \propto e^{\sigma \ln N_t - m \int_0^t \ln N_\tau \, d\tau}.$$

[11]See the authors' footnote 13 and the notation following their equation 17.

[12]Beyond rearranging the expression of the model, we have simplified it by (i) eliminating a constant (the original would have ...$(\ln(S_{it}/f) - \mu)$ for $S_{it} \geq f e^\mu$), (ii) using A_{it-1} in place of Young's "λ_i(max)" construction, and relatedly (iii) removing the possibility of choosing $A_{it} <$ "$\overline{\lambda}(t-1)$". These simplifications have no bearing on how shrinking variety fails to deliver infinite utility or output.

[13]That is, we are normalizing the initial research population to 1 and taking no advantage of the fact that the research population may exhibit sustained growth if $\gamma > 0$.

For the exponent to grow at least exponentially, its derivative

$$\sigma \dot{\ln N_t} - m \ln N_t \tag{15}$$

must also. If $-\ln N$ grows superexponentially, $\dot{\ln N}$ grows more quickly in absolute value and (15) tends to negative infinity. If it grows subexponentially, A is not double-exponential. The fastest growth path is therefore double-exponential. This is achievable if $\ln N$ grows exponentially at rate $n < m/\sigma$, so that (15) grows like

$$(m - \sigma n)\big(-\ln N_t\big) > 0.$$

Infinite utility is therefore achievable, given a logarithmic utility function, iff $\gamma - \rho < m/\sigma$.

Efficiency in the Young model — The Young model proper avoids extreme inefficiency for all parameter values because the discreteness of time amplifies the diminishing returns. Letting g_{it} denote the instantaneous exponential growth rate of process efficiency realized over the course of one period, we have

$$\begin{aligned} e^{g_{it}} &= \frac{A_{it} - A_{it-1}}{A_{it-1}} \propto \ln\left(\frac{S_{it}}{f}\right) \\ \implies g_{it} &\propto \ln\left(\ln\left(\frac{S_{it}}{f}\right)\right). \end{aligned}$$

In the context of the rest of the model, this is enough to guarantee not only that exponential growth is *not extremely* inefficient but that it is not inefficient at all. The optimal growth rate is greater than the equilibrium growth rate—horizontal innovation keeping N proportional to L is supplied at the efficient rate, vertical innovation is undersupplied—but constant (see Young's Section IV).[14]

The intuition for the optimality of $g_N = \gamma$, given sufficiently severe diminishing returns to research labor, is clearest if we imagine the extreme case:

$$\frac{\dot{A}_{it}}{A_{it}} \propto \begin{cases} g, & S_{it} \geq f; \\ 0, & S_{it} < f. \end{cases}$$

3 Lab equipment models

This section discusses inefficiency in the lab equipment SWE literature: Aghion and Howitt (1998, ch. 12.2), Howitt (1999), Howitt (2000), Peretto (2018), Aghion et al. (2025),

[14] An earlier working paper version, Young (1995), also considers a model in which love of variety comes via a Salop circle rather than a Dixit-Stiglitz aggregator. This model avoids the inefficiency results similarly: the research function is unchanged, and the consumption aggregator does not tend to 0 with N slowly enough to make arbitrarily little variety desirable. Also, in this model N is discrete, so consumption is positive only if $N \geq 1$.

and Massari and Peretto (2025). On the whole, the conclusions reached in the research labor context are maintained. For simplicity, we will not specify preferences and focus only on how the models generate Result 2, trusting that the applications to Result 1 are straightforward.[15]

Benchmark: Peretto (2018) (simplified) — Output equals

$$Y_t = \left(\int_0^{N_t} Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1; \tag{16}$$

$$Y_{it} = A_{it} L_{it}, \tag{17}$$

so that, given symmetric technology $A_{it} = A_t$ ($i \leq N_t$) and an efficient allocation of labor to each good,

$$Y_t = A_t N_t^{\sigma} L_t, \ \ \sigma \equiv \frac{1}{\varepsilon - 1} > 0. \tag{18}$$

As before, assume for simplicity that this symmetry holds at time 0. Assume also that the population L grows at rate $\gamma \geq 0$.

Process efficiency for good i grows according to

$$\dot{A}_{it} \propto P_{it} \tag{19}$$

where P_i denotes the rate of investment, in units of *output* rather than labor, into i's process efficiency.[16]

Letting $P_i/(YN) \in (0, 1)$ be fixed and equal across i, so that the [process] research share

$$p \equiv \int_0^N P_i di \Big/ Y$$

is constant and this investment is spread equally across goods, we have

$$\dot{A}_t \propto Y_t/N_t \propto A_t N_t^{\sigma-1} L_t. \tag{20}$$

If $\sigma < 1$, then with $\lambda \equiv 1 - \sigma$, Result 2 applies immediately.

If $\sigma \geq 1$—i.e. $\varepsilon \leq 2$—vertical innovation does not accelerate when variety shrinks. In this case, when variety doubles, output weakly more than doubles, so even lab equipment *per good* weakly increases. Variety is unambiguously desirable. Precisely for this reason, however, the SWE mechanism for defusing superexponential growth fails. Given $\gamma > 0$, output per person grows double-exponentially, not only on a feasible path attainable by

[15] Incidentally, unlike in the research labor SWE literature, utility in consumption is not universally assumed to be logarithmic, so double-exponential growth does not necessarily have qualitatively extreme welfare implications.

[16] We are using A to denote Peretto's "Z^{κ}", but here considering only the $\kappa = 1$ case. The $\kappa > 1$ case is discussed below.

fixing N but in equilibrium. For this reason, Peretto restricts his attention to the $\sigma < 1$ case.[17]

Note from (19) that the rate of vertical innovation is assumed to be linear in investment. That is, in the notation of the previous section, it is assumed that $\lambda = 1$. This is because here, technology supports technological development only via the channel that $Y \propto A$ and $\dot{A}_i \propto S_i \propto Y$. If the function from P_i to A were not linear, the model would not satisfy the central desideratum that, fixing L and N, output grows exponentially through vertical innovation.

Peretto (2018) proper and Massari and Peretto (2025) — The only relevant difference between Peretto (2018) proper / Massari and Peretto (2025) and the simplification above is that, in place of (19), they use[18]

$$\dot{A}_{it} \propto A_{it}^{\zeta} P_{it},$$

so that, fixing $P_i/(YN) > 0$,

$$\dot{A}_t \propto A_t^{1+\zeta} N_t^{\sigma-1} L_t.$$

The $\zeta > 0$ case yields a result more extreme than in any model yet: even fixing rather than shrinking N, process efficiency and output can grow hyperbolically. (Output grows exponentially in equilibrium because the incentive structure leads profit-maximizing firms to develop new goods at the rate needed to offset the growth contributions both of ζ and of population growth.) Result 2 is maintained trivially.

Likewise, in the $\zeta < 0$ case, output grows exponentially in equilibrium because the market structure leads profit-maximizing firms to develop goods slowly enough (or abandon them quickly enough) to make up for the drag on growth imposed by ζ net of population growth.[19] Positive growth is feasible iff $1 - \sigma(1 - \zeta) > 0$, but when this condition holds, so does Result 2, with $\lambda \equiv 1 - \sigma(1 - \zeta)$.

Note that in the $\zeta < 0$ case, if $\gamma = 0$ exponential growth requires $N_t \to 0$. Negative variety growth in this model is therefore explicitly feasible.

Howitt (1999) — Output equals

$$Y_t = \int_0^{N_t} Y_{it}^{1-\sigma} di, \ \sigma \in (0, 1);$$
$$Y_{it} = A_{it}^{\frac{1}{1-\sigma}} L_{it}.$$

[17]See p. 54, especially the equivalence to the Romer model given $\sigma \geq 1$ discussed in footnote 6.

[18]After the change of variables noted in footnote 16 above, our ζ is Peretto [and Massari]'s $1 - 1/\kappa$.

[19]This case is covered only by Massari and Peretto (2025). Like Young (1998) and Peretto and Connolly (2007), this paper features fixed costs per period to producing a good, rather than the more typical one-time costs of developing a new good, so as to maintain symmetry between growing and shrinking N. As noted in the discussions of these papers above, fixed costs per period only strengthen the inefficiency results.

Given symmetric technology $A_i = A$ $(i \leq N)$ and an efficient allocation of labor $L_i = L/N$ $(i \leq N)$, this would yield

$$Y_t = A_t N_t^{\sigma} L_t^{1-\sigma}.$$

Implicitly, output exhibits constant returns to scale in (i) the Dixit-Stiglitz aggregate (16) of intermediate goods i and (ii) a fixed factor whose share coincides with σ. Note that A_i no longer precisely denotes i's process efficiency, but that by construction output remains linear in the (weighted) average A of the $\{A_i\}$.[20]

Howitt assumes that goods' process efficiencies grow stochastically, so the $\{A_i\}$ are not equal. As in Dinopoulos and Thompson (1998), however, the process is such that, given equal research investment per product, the shape of the distribution is time-invariant.[21] The efficient labor allocation then maintains

$$\int_0^N Y_i^{1-\sigma} di \propto N \cdot \left(A^{\frac{1}{1-\sigma}} L/N\right)^{1-\sigma},$$

and thus

$$Y_t \propto A_t N_t^{\sigma} L_t^{1-\sigma}.$$

As in Peretto (2018), process efficiency for good i grows essentially according to (19). The only difference is that $\dot{A}_i$ is linear not in P_i but in $P_i \cdot A_i / \max_i(\{A_i\})$, but by invariance the numerator and the denominator grow at the same rate over the long run. The aggregate result, fixing the research share s, is a deterministic process familiar from (20):

$$\dot{A}_t \propto Y_t/N_t \propto A_t N_t^{\sigma-1} L_t^{1-\sigma}.$$

Because $\sigma < 1$, Result 2 follows immediately.

Aghion and Howitt (1998, ch. 12.2) and Howitt (2000) — Output equals[22]

$$\begin{aligned} Y_t &= N_t^{\alpha-1}\Big(\int_0^{N_t} Y_{it}^{\alpha} di\Big) L_t^{1-\alpha} \cdot \left(\frac{N_t}{L_t}\right)^{\sigma}, \ \alpha \in (0,1), \ \sigma \in [0,1); \qquad (21) \\ Y_{it} &= A_{it}^{1/\alpha} K_{it}, \end{aligned}$$

where the capital stock $K = \int_0^N K_i di$ grows with saved output in the usual way, but this will not be relevant for our purposes.

[20]For consistency with the other sections, our σ is Howitt's $1-\alpha$. Also, Howitt defines A_i so that Y_i is linear in A_i but Y is nonlinear in average process efficiency; we use a change of variables to do the reverse.

[21]Asymptotically or, if the initial distribution is the asymptotic distribution, precisely.

[22]For consistency with the other sections, we have used σ for the authors' β and let Y_i stand for the authors' $A_i^{1/\alpha} x_i$. We have then redefined A_i for the reasons in footnote 20 above.

As in Howitt (1999), process innovations arrive stochastically, but in a way that maintains an invariant distribution of $\{A_i\}$ and, in the efficient capital allocation,

$$\int_0^N Y_i^\alpha di \propto N \cdot \left(A^{1/\alpha} K/N\right)^\alpha = N^{1-\alpha} A K^\alpha$$
$$\implies Y_t \propto A_t K_t^\alpha L_t^{1-\alpha} \cdot \left(N_t/L_t\right)^\sigma,$$

where A as usual may be an average or weighted average of the $\{A_i\}$.

Remarkably, in the model studied throughout the body of the textbook chapter and the entirety of Howitt (2000), the $(N/L)^\sigma$ term at the right of (21) is absent; σ equals zero. The $N_t^{\alpha-1}$ term at the left precisely nullifies the benefits of variety that the Dixit-Stiglitz-style aggregate is usually designed to capture, and inventing new goods effectively just fragments "A" into smaller pieces. In footnote 6 of the chapter, the authors acknowledge that "[l]iterally, the model implies that the optimal number of different products is vanishingly small", but say that this "counterintuitive welfare result" can be eliminated with the tweak above.[23]

In the case of Howitt (2000), this is always false. Given a constant research share p and an equal allocation of process efficiency investment across goods, A evolves according to

$$\dot{A}_t \propto Y_t/N_t$$

for the usual reasons. By assumption $\sigma < 1$, so Result 2 follows at once.[24]

In the chapter itself, A evolves according to

$$\dot{A}_t \propto A_t R\left(\frac{pY_t}{A_t N_t}\right) \text{ with } R(0) = 0, R' > 0, R'' < 0$$
$$= A_t R\left(pK_t^\alpha L_t^{1-\alpha-\sigma} N_t^{\sigma-1}\right). \tag{22}$$

From the discussion of Peretto (2018) proper, we know that if R' diminishes no more steeply than a power function, Result 2 still follows. As in the continuous-time analog to Young (1998), Result 1 holds—more precisely, double-exponential growth is feasible—unless $R(\cdot)$ is sublogarithmic.[25]

[23]They argue that placing an L in the denominator, so that doubling population and variety has no impact on output per person, "might be justified by the fact that the variety of different tastes... expands as people become more numerous" or by "thin-market costs". Removing this penalty to population (or making it weaker than σ) would affect the equilibrium, but it has no bearing on the inefficiency results.

[24]In fact, the paper assumes for simplicity that utility is linear in consumption, so even a drop in variety big enough to raise the growth rate above the discount rate generates infinite utility.

[25]Though capital accumulation now contributes to output and, indirectly, to technological development, this does not change the qualitative result. To see this, observe that with Y growing double-exponentially, K grows less than double-exponentially, so by (22) with $R(x) = \ln(x)$ and $N_t \propto e^{-e^{nt}}$, g_A still grows exponentially. The rest of the proof sketch in Section 2.2 follows.

Aghion et al. (2025) — Output equals (16)–(18), as in Peretto (2018). Process efficiency for good i grows according to[26]

$$\dot{A}_{it} \propto A_{it}\Big(P_{it}A_t^{\frac{1-\sigma}{\sigma}}A_{it}^{-\frac{1}{\sigma}}L_t^{-\sigma}\Big)^{\lambda},\ \lambda > 0, \tag{23}$$

where A is a CRS, CES aggregate of $\{A_i\}$ with elasticity of substitution greater than 1.[27] As usual, assume symmetry, replace A_i with A, and, fixing p, substitute Y/N for P_i to get

$$\dot{A}_t \propto A_t\big(L_t/N_t\big)^{\lambda(1-\sigma)}.$$

Just as in Peretto (2018) [with $\zeta = 0$], Result 2 holds iff the exponent on N is negative, which it is iff $\sigma < 1$ ($\varepsilon > 2$).

Recall that in Peretto (2018) with $\sigma > 1$ ($\varepsilon \in (1, 2)$), the rate of vertical innovation increases in both population and in variety, since a growing number of varieties grows output so quickly that even output per variety increases. An expanding range of varieties thus fails to defuse the double-exponential growth that obtains when variety is fixed, and this motivates a restriction to the $\sigma < 1$ case. Here, by contrast, if $\sigma > 1$, the exponent on (L/N) is *negative.* As a result, the $\sigma > 1$ case does not exhibit extreme inefficiency (even though the model uses log utility): double-exponential growth would require exponential growth in N/L, rather than L/N, and the costs of developing new goods (which appear in the paper as roughly equivalent to labor costs) render the former infeasible. In short, when $\sigma > 1$, Aghion et al. overturn the relationship from growing populations to growing growth rates by positing outright that, in this case, *the maximum feasible rate of vertical innovation decreases in population per product.*[28]

This illustrates that extreme inefficiency is avoided (under some parameter values) if we go beyond sharply decreasing returns to vertical innovation, and assume negative returns (under said parameter values). More precisely, the assumption is that the existence of human beings generates a direct negative contribution to vertical innovation, which can be relieved only by creating variety. This direct cost is present in the model for all parameters, and is stipulated to have elasticity σ, as can be seen from (23). If $\sigma > 1$, this negative contribution grows more quickly in L/N than the indirect benefit: namely that a larger population creates, with elasticity 1, more output to devote to research. The authors justify the modeling choice that places "$L^{-\sigma}$" in (23) as "captur[ing] the effect that

[26] This follows from rearranging the authors' equation 6, using ε for their θ (so our σ equals their $1/(\theta - 1)$) and λ for their $1/(1 + \zeta)$. They focus only on the $\lambda < 1$ ($\zeta > 0$) case.

[27] As with Dinopoulos and Thompson (1998), we posit that N may shrink, so that if research and production are restricted to goods up to N_t, A_t equals $\{A_{it}\}$ ($i < N_t$). Here, however, the paper is explicit that exit is possible and that discontinued goods are removed from the technology aggregate.

[28] Another distinctive feature of the paper is that it posits that a stream of investment is needed to keep a given range of goods "alive", somewhat analogous to the fixed costs of Peretto and Connolly (2007), Young (1998), or Massari and Peretto (2025). This is the motivation for the paper: the fixed costs are interpreted as the costs of developing new goods once we run out of further process efficiencies for the old goods. These costs do not change the conditions under which Result 2 obtains, but as in the above cases, because a smaller good range requires smaller fixed costs, they only strengthen the result.

a larger market leads to faster economic growth", but it is unclear how it could capture this effect, since it is in fact a stipulation that larger markets make growth more costly. It is also unclear why we should expect the elasticity of this cost to be precisely σ, so that population growth puts a net drag on technological development precisely when it would otherwise suffice to render growth double-exponential.

In any event, when calibrating the model, the authors use a conventional estimate of $\varepsilon = 4$, or $\sigma = 1/3$ (see Table 1).

4 Discussion

We have seen that the SWE approach to growth theory implies either (i) that vertical innovation faces severely diminishing returns or (ii) that explosive growth is feasible but goes unrealized due to a colossal market and/or policy failure. This "dilemma" has implications for the plausibility of the SWE approach. If we accept that the approach is broadly accurate nonetheless, the dilemma in turn has significant implications for the growth impacts of automation.

4.1 Implications for second-wave endogenous growth theory

Plausibility of severely diminishing returns — The SWE literature since Young (1998) has entirely abandoned the assumption of sublogarithmic returns to investments in process efficiency.[29] Indeed, even Young does not frame his research function as sublogarithmic explicitly. The most common assumption is that the returns are linear. This is presumably because such steeply diminishing returns are considered intuitively implausible and because firm-level studies of the elasticity of productivity increases to research investment do not typically find them.[30]

That said, it does not seem unreasonable to suppose that the elasticity quickly approaches zero outside the observed range. Perhaps no research team, however large, could double the process efficiency of a typical assembly line in five minutes.

Plausibility of extreme inefficiency — The alternative possibility is that explosive growth is and has long been technologically feasible. This possibility may be highly counterintuitive, but if it is true it is among the most important facts of all time. It is worth evaluating seriously.[31] Market and policy failures can destroy markets even in the developed world,

[29] With the partial exception of Aghion and Howitt (1998, ch. 12.2), as noted.

[30] See e.g. Hall and Mairesse (1995), Lanjouw and Schankerman (2004), Klette and Kortum (2004), and Akcigit and Kerr (2018). To be sure, this may be largely because it is standard practice not to look for them, and only to estimate a local elasticity.

[31] Indeed, one of the primary motivations for the SWE approach to growth theory is that it is said to match the evidence on how policy can affect the growth rate: see the introduction of Aghion et al. (2025) for an up-to-date statement of the case. If this is true, its implication that a new policy paradigm could send growth to infinity is worth taking especially seriously.

and have kept growth rates low in some developing countries for decades even after many examples of rapid catch-up growth have offered case studies for others to follow. Ultimately, however, this possibility appears untenable for at least three reasons.

First, if explosive growth is feasible, the size and speed of its benefits drastically exceed those of resolving any other case of stagnation or market failure. A country may stagnate if a small increase to its growth rate would require investments or policy changes with large up-front costs, especially if the costs accrue to its current elite (see e.g. Acemoglu and Robinson (2012)), but any such costs will pay for themselves if the growth impact is sufficiently large and rapid. Moreover, the history of Mao's China demonstrates that it is feasible for a modern state to enforce severe "negative horizontal innovation", restricting output to a narrow range of goods. History also of course offers many examples of large, successful, publicly sponsored research projects in narrow domains, such as the Manhattan Project, the Apollo Program, or the Human Genome Project. It is hard to see why no state would be able to afford whatever costs are associated with sustaining negative horizontal innovation and a large research program at the same time.

Second, the logic of an SWE model implies (absent sublogarithmic returns to investments in process efficiency) that, even if a single firm or community refused to trade with the rest of the world, it would be able to generate explosive growth on its own. This putatively does not happen only because the members of any such group would face a constant temptation to depart and establish monopoly over a new product, whose rents would be astronomical due to its complementarity to the few but plentiful products produced by the rest. The solutions to this kind of coordination problem are straightforward: e.g. a consortium agreement ensuring that, if any party withdraws, the project is canceled. It is hard to imagine that no group in the world, over generations, could have implemented one.

Third, many people already choose careers in part on the basis of their social impact. Many more would accept below-market pay to join a team—indeed, surely many would pay to join a team—that could open the gates of heaven.

Alternative preferences — If we reject both the possibilities above, we reject the SWE literature, at least in its current form. The most common alternative, as noted in the introduction, is semi-endogenous growth theory. Semi-endogenous growth models maintain the Dixit-Stiglitz preference specification, but propose an alternative research function which implies that economic growth can be sustained only by population growth.

Another possibility is that the SWE-style research function (on both dimensions) is roughly accurate, but that the Dixit-Stiglitz aggregator leads us badly astray. In particular, if marginal utility diminishes rapidly enough in the consumption of each good, consuming a wider range of goods is sometimes preferable even to arbitrarily large quantities of a narrower range of goods. Then even if explosive growth in a vanishing range of goods is feasible, as in Result 2, it may be preferable to widen the range of goods instead.[32] In

[32]Work in progress with Chad Jones explores this and related points. For a very early draft containing

combination with fixed labor costs to maintaining a given product line, it may be optimal to keep variety proportional to population, such that stagnant populations in the long run engage only in vertical innovation.[33]

Because the required preferences are highly nonhomothetic, however, they do not admit a consumption aggregate at all. There is then no straightforward sense in which economic growth has historically been exponential, the central stylized fact which SWE and semi-endogenous models both seek to explain. Though an alternative preference specification may salvage parts of the SWE approach, therefore, it should arguably motivate a reexamination of growth theory more fully.

4.2 Implications for growth after automation

Severely diminishing returns — In a semi-endogenous model, economic growth proceeds at roughly its maximum possible rate. Growth is, in a technological sense, more difficult than in an SWE model: it is constrained not by equilibrium decisions to expand variety but simply by a lack of people. Precisely for this reason, however, a semi-endogenous model predicts that in the event of full automation—where output can be turned into robots capable of performing every task in research and production—growth will accelerate dramatically. In fact, even sufficient partial automation renders growth hyperbolic.[34]

Now consider a simple SWE-style model, under full automation, in which vertical innovation faces severely diminishing returns. Consumption good i is produced by the "bots" B_i allocated to producing it, which they do with efficiency A_i:

$$c_{it} = A_{it} B_{it},$$

so that under symmetry the Dixit-Stiglitz aggregator equals

$$c_t = A_t N_t^{\sigma} B_t,$$

where $\int_0^N B_i di = B$ and as usual $\{A_i\} = A$. Fixing the share of robots engaged in vertical innovation, process efficiency for good i grows according to[35]

$$\dot{A}_{it} = A_{it} R\big(B_t / N_t\big) \text{ with } R' > 0, \lim_{x \to \infty} R(x) = \bar{g}.$$

Like consumption goods, bots are themselves a produced good. Fixing the shares of bots engaged in bot production and bot vertical innovation, the bot population grows

some of the intuition, see Trammell (2024).

[33] I thank Pete Klenow for this point.

[34] See Aghion et al. (2019), Section 4.1, example 3. For a corrected proof see Trammell and Korinek (2023), Section 6.

[35] Positing that $R(\cdot)$ is upper-bounded simplifies the analysis. Unbounded but sufficiently concave research functions can likewise yield the conclusion below that, on an efficient path, output grows double-exponentially instead of exhibiting a vertical asymptote.

according to

$$\dot{B}_t \propto A_{\text{BOT},t} B_t,$$
$$\dot{A}_{\text{BOT},t} = A_{\text{BOT},t} R(B_t).$$

We ignore depreciation because, with B growing superexponentially, the exponential depreciation process is irrelevant in the limit. We stipulate that production and research are done by robots rather than by undifferentiated output to distinguish increases in consumption due merely to a taste for variety, i.e. those induced by raising N, from increases in productive capacity that allow robots themselves to accumulate more quickly.

In the limit, A grows at a rate no greater than $\bar{g}$ and B grows double-exponentially at a rate proportional to $e^{\bar{g}t}$. The final source of consumption growth is then g_N.

Fix the share of bots engaged in developing new goods. Suppose that new goods inherit average process efficiency, as roughly in the existing SWE literature (so that the technology aggregate A grows like A_i for each i in production, as stipulated). If, by analogy to one strand of the literature,

$$\dot{N}_t \propto B_t,$$

then N grows like an exponential integral—slower than double-exponentially. The assumption that "new goods per bot" is constant might be justified on the grounds that developing a new good j and a process for it that is as efficient as the existing A grows more difficult as A rises, in a way that just offsets whatever positive spillovers come from the technology associated with high process efficiency in existing goods. Alternatively, following another strand, we might simply specify

$$N_t \propto B_t.$$

Then N grows double-exponentially with B. In either case, c grows at most double-exponentially like $e^{e^{\bar{g}t}}$.

In this framework, we know that $\bar{g}$ exceeds the current rate of process efficiency growth (say 1% per year, if the economic growth rate is roughly split between vertical and horizontal innovations). We also know that $R'(x)$ begins to diminish rapidly for x not too far above $R^{-1}(1\%)$, or else a result qualitatively similar to Result 2 would obtain; the burst of growth achievable by shrinking variety would be finite but dramatic. It is difficult to say much more. In any case, double-exponential growth is eventually arbitrarily rapid—we are imposing no limits on the rate at which robots will ultimately be able to self-replicate or build the things we value—but there is a big difference between a vertical asymptote and a growth rate that itself grows at, say, 10% per year.

Note how the central mechanism behind the result, an extremely concave research function, would be incompatible with the semi-endogenous framework (without further modifications). For exponentially growing research labor to drive exponential growth in $\dot{A}$, $R(\cdot)$ can be at worst power-functional.

Extreme inefficiency — Briefly, suppose we accept the SWE framework but reject the conclusion that vertical innovation faces severely diminishing returns, maintaining instead that growth has historically been extremely inefficient because of the coordination failures discussed in the previous subsection. Then automating process efficiency research, even for a narrow band of goods, yields Result 2 as long as the automated researchers can be instructed to coordinate.

Unlike in a semi-endogenous framework, hyperbolic growth does not require the automation of any part of production. The "bots" do not have to be numerous, self-replicating, or able to produce any of the final goods we value.

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