Ethical Consumerism

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1 Introduction

Agents may have preferences not only over the quantities of goods they themselves consume but also over the total quantities of various goods supplied.

Consider a consumer with concern for animal welfare. The consumer's utility is increasing in his own meat consumption, but decreasing in total meat consumption: in the consumption of pigs and chickens in particular. How he should adjust his purchasing behavior, relative to what his optimal purchases would be if he had no concern for animal welfare, is not obvious. If he believes that the production of a dollar's worth of chicken creates more misery than the production of a dollar's worth of pork, for instance, he may naively be inclined to prioritize reducing his purchases of chicken over reducing his purchases of pork. If the supply of pork is more price-elastic and demand for pork less price-elastic than that of chicken, however, this inclination may be misguided. Buying less chicken in this case simply causes the price to fall and the quantity demanded by other consumers to rise, with little net impact on the quantity of chicken consumed. Buying less pork, by contrast, generates a substantial decrease to the quantity of pork consumed. Cutting back on pork may thus be the higher priority.

Complicating matters further, however, our consumer must consider the impact of his purchases of a good not only on the quantity of that good, but on the quantities of all the goods he cares about: here, that is, on the quantities of both pork and chicken. If buying less chicken causes other consumers to substitute to chicken from pork, then cutting back on chicken may be the best policy after all.

The purpose of this paper is to characterize, in light of these complications, equilibrium market behavior by "ethical consumers" in a competitive production economy. We will work in a static setting and assume that all parties have complete information.

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Bergstrom et al. (1986) study equilibrium spending behavior by public good providers in light of crowd-out issues like those discussed above. The model we will consider is conceptually related to Bergstrom et al.'s model, but (a) we will allow the cost of providing each good to be determined endogenously by the starting endowments, firm production functions, and quantities of other goods purchased; (b) our economy will be populated not by individuals not all of whom care about goods' supply levels as opposed to their own consumption of them; and (c) we will assume that all agents are "atomic", in the sense that they can act as price-takers for some purposes.

Dufwenberg et al. (2011) purport to study "other-regarding preferences in general equilibrium". In particular, they present a model intended to capture equilibrium purchasing behavior in a static production economy in which individuals may have preferences defined not only over their own consumption baskets but also over the consumption baskets, utility levels, or budget sets achieved by other consumers. If they had in fact captured equilibrium purchasing behavior in such a setting, their project would have encompassed the project of the present paper, since concern for the total production of various goods is a special case of concern for others' consumption baskets. Remarkably, however, Dufwenberg et al. maintain the classical assumption that each individual is *fully* "atomic", in the sense that her own purchases have no impact on either prices *or aggregate production*. They thus conclude that an individual's other-regarding preferences have no impact on her purchasing behavior, unless they interact nonlinearly with her self-regarding preferences: for example, if she prefers brown bread to white bread if and only if her neighbor has two cars. A similar approach to other-regarding preferences, or preferences over total production levels, is taken by Kreps (1990) (p. 203), Ellickson (1994) (§7.3), Sobel (2009), and many others.

In fact, however, though a consumer's ability to influence the price of a good vanishes as the surrounding economy grows, any such price change slightly influences more purchases by others. An ethical consumer's concern for the externalities of his purchases in a competitive economy is thus entirely consistent with the intuitive and well-known result (Roberts and Postlewaite, 1976) that, as the surrounding economy grows and individual market power falls, self-interested consumers are incentivized to act ever more like price-takers. If buying one unit of chicken induces, in equilibrium, the production of 0.6 more units of chicken in an economy with one billion participants, it also induces the production of approximately 0.6 more units of chicken in a "doubled" economy with two billion participants but an identical distribution of endowments and production technologies. The size of an "ethical externality" in this sense does not in general fall to zero as an economy approaches perfect competition.

Finally, Wilkinson (2022) argues that consumers are morally obligated to account for the market externalities of their purchases: i.e. the impacts that their purchases have on prices, and the impacts of these price-shifts on others' purchases and ultimately others' welfare. However, in his illustrations, he only calculates the partial equilibrium impacts of a given choice of purchasing behavior. In effect, he assumes that cross-price elasticities of demand and supply are zero, at least across goods that a consumer holds to have nonzero ethical externalities. He also focuses on the case in which individuals have preferences over the equilibrium income levels of other individuals, rather than the (as we will see) strictly more general case in which individuals have preferences over good supply levels. The analysis presented here fills these gaps.

2 Model

There are L goods.

The production set is denoted Y. We will assume that it contains the origin and is convex, closed, and bounded above with a continuously differentiable and strictly concave upper boundary.

There is a large number I of individuals, with starting endowments $\omega^i \neq 0$ (or $\omega^i_\ell \geq 0$ of good ℓ), and utility functions u^i for $i = 1, ..., I$. The vector of total initial endowments is ω, and the total initial endowment of good ℓ is ω_ℓ. Likewise, the vector of net production quantities is $y \in Y$, and the net production of good ℓ is y_ℓ.

Individual i's vector of purchases is denoted x^i. The vector of total purchases of each good is denoted x. Purchases are made according to price vector p.

Though we will study the equilibrium properties of this economy in a static setting, we may for clarity imagine production and trade unfolding over the course of four steps. First, each individual i chooses an L-length vector ψ^i, which might be interpreted as a set of good-specific "ethical weights". (ψ denotes the $L \times I$ matrix constituting all individuals' good-specific ethical weights.) Second, each individual i sells her endowment, earning an income of $p \cdot \omega^i$, and goods are processed according to the production vector $y(p)$ that maximizes aggregate profits π, as defined by (7) below. (Note that $y(p)$ is differentiable, by the differentiability of the upper boundary of Y.) Third, each individual i receives a share θ^i of π. Finally, given y and ψ^i, each i purchases $x^i \geq 0$ to maximize what we will call her quasi-utility function $\tilde{u}^i$ subject to her budget constraint.

The idea may be reframed as follows. People strategically submit (binding) demand functions to the Walrasian auctioneer. An individual's demand function is not fully determined by her budget and the price vector, but also by how a given submitted demand function will affect total production in the resulting equilibrium (which depends on what demand functions others are submitting). In particular, the demand function i submits is that which would be implied by a strategically tweaked version of i's true utility function $u^i(\cdot)$, namely $\tilde{u}^i(\cdot)$. The strategic tweaks in question are determined by ψ^i.

The vector of total quantities of each good in existence after production has taken place will be called supply, and it will be denoted $s \triangleq y + \omega$ (≥ 0).

For each i, quasi-utility function $\tilde{u}^i(\cdot)$ takes the form

$$\tilde{u}^i(x^i, \psi^i) = v^i(x^i) + \psi^i \cdot x^i, \tag{1}$$

defined over positive (or, depending on $v^i(\cdot)$, perhaps nonnegative) values of x^i and all values of ψ^i. We will call $v^i(\cdot)$ i's felicity function and, as noted above, ψ^i i's vector of ethical weights.[1]

The agent's utility function $u^i(\cdot)$ represents her all-things-considered preferences, defined over x^i and s. We will assume that it has a representation additively separable across x^i and s, taking the form

$$u^i(x^i, s) = v^i(x^i) + w^i(s), \tag{2}$$

where both $v^i(\cdot)$ and $w^i(\cdot)$ are continuously differentiable (noting in particular that $\nabla w^i(s)$ is finite at all $s \geq 0$), and where $v^i(\cdot)$ is increasing and satisfies the Inada condition that

$$\lim_{x^i_\ell \to 0} \frac{\partial v^i(x^i)}{\partial x^i_\ell} = \infty \;\; \forall \ell. \tag{3}$$

We will assume that there is at least one individual whose preferences are defined only over own consumption. More formally,

$$\exists i : w^i(s) = 0 \;\; \forall s. \tag{4}$$

We will also impose one more technical assumption on the $\{w^i(\cdot)\}$, noted below.

Given quasi-utility functions $\tilde{u}_i(\cdot)$ as in (1), we can define demands as a function of prices, ethical weights, and profits:

$$x^i(p, \psi, \pi) = \arg\max_{x^i} \Big(v^i(x^i) + \psi^i \cdot x^i \Big) \;\Big|\; p \cdot x^i \leq p \cdot \omega^i + \theta^i \pi, \tag{5}$$

$$x(p, \psi, \pi) = \sum_i x^i(p, \psi, \pi). \tag{6}$$

Our assumptions straightforwardly imply that demand functions are well-defined, i.e. that demand correspondences are single-valued. It is not as straightforward that they imply that demand functions exhaust individual budgets. This will be shown in §3.

[1] These terms are intended only as concise ways to distinguish $v^i(\cdot)$, used to represent the preferences i has over the quantities of goods she herself purchases, from ψ^i, used to represent the preferences i has over total supply levels. The former preferences will often consist primarily of preferences for individual pleasure or wellbeing, and the latter will often consist primarily of ethical preferences, but there is no necessary connection. Also, note that the usage of the term "felicity function" here bears no close connection to its occasional usage in the intertemporal choice literature as an alternative to "flow utility function".

Given prices, ethical weights, and production functions, aggregate profits equal

$$\pi(p, \psi, y) = \pi : p \cdot x(p, \psi, \pi) = p \cdot (\omega + y). \tag{7}$$

Note that

$$\begin{aligned} p \cdot x(p, \psi, 0) &\leq p \cdot \omega \quad (8) \\ &\leq p \cdot (\omega + y(p)), \quad (9) \end{aligned}$$

where (8) follows from (5). Also, by (4), $p \cdot x(p, \psi, \pi)$ increases without bound in π. Thus, as long as $y = y(p)$, there is thus a unique $\pi \geq 0$ satisfying (7). (As explained below, $y(p)$ is the unique production vector maximizing $p \cdot y$, and this will also be the unique production vector maximizing π.)

This definition of aggregate profits is a standard alternative to the more conventional $\pi(p, y) = p \cdot y$ used in the event that individuals are not locally nonsatiated.[2] Here, we cannot assume local nonsatiation: we must allow for the possibility that, in some circumstances and for some preferences individuals might have about aggregate supply levels, an individual maximizes his utility by spending less than all his budget. Defining profit to equal $p \cdot y$ would imply that, even in equilibrium, more-than-satiated consumers would give rise to $p \cdot x < p \cdot s$. Firms would in effect remain in possession of unsold goods of value of $p \cdot (s - x)$, leaving them to rot on the shelves.

Definition (7), by contrast, amounts to the more natural and realistic assumption that any such excess production would belong to the producer, and thus to individuals according to shares θ. This implies that if $p \cdot x(p, \psi, \pi) < p \cdot s(p)$, profits π are incompatible with (p, ψ). Given (p, ψ), aggregate profits in fact exceed π by $p \cdot (s - x)$. Likewise, if $p \cdot x(p, \psi, \pi) > p \cdot s(p)$, profits π are incompatible with (p, ψ) because they imply that consumers buy goods worth more than all the goods available to sell. In this case at least some of the dividends returned to consumers were not true profits at all; firms would have had to withhold at least some of these profits in order to fill the orders given by x.

Observe that when individuals do exhaust their budgets,

$$p \cdot x(p, \psi, p \cdot y) = p \cdot \omega + p \cdot y = p \cdot s. \tag{10}$$

Profits then equal $p \cdot y$, as usual. Also, even in the more general setting of (7), because x is increasing in π, assuming that production $y(p)$ maximizes profits is equivalent to assuming that production maximizes $p \cdot s$ and thus $p \cdot y$.

Finally, for simplicity, let

$$x(p, \psi) \triangleq x(p, \psi, \pi(p, \psi, y(p))), \tag{11}$$

[2] Konovalov (2005) offers an review of the literature using this approach, at least as of 2005. The resulting equilibria are called "dividend equilibria" or "Walrasian equilibria with slack".

where $\pi(\cdot)$ is defined as in (7).

Before proceeding, let us make three clarifications.

First, it may seem as though the behavior described by maximizing (1) amounts to assuming that one's own purchases have no impact on total production, and simply associating the consumption of each good ℓ with some "ethical" contribution to one's own utility that can be represented as growing linearly in the consumption of the good. This is not the behavior being modeled, however. Rather, we are assuming that i chooses ψ^i, and ultimately x^i, entirely so as to maximize $u^i(x^i, s)$ in equilibrium. In a single-period setting, because purchases are made after production, it is true that choices of x^i cannot affect s. In equilibrium, however, s is determined by y, which is determined by p, which is set in turn by individuals' demand functions and thus in part by ψ. i's choice of ψ^i therefore marginally impacts s. As we will see, for an appropriate choice of ψ, the policy of making purchases so as to maximize the quasi-utility function (1) in fact maximizes (2) in equilibrium.

Second, it may seem as though individuals consider their impact on total production only in their purchasing choices and not in their selling choices. That is, it may seem as though individuals are assumed to maximize their income by selling the entirety of their endowments, regardless of the extent to which their endowments are used in the production of goods they consider to have negative externalities. In fact, however, refusing to sell part of one's endowment ω^i_ℓ is here equivalent to increasing one's purchases of x_ℓ. If such purchases would produce a favorable marginal effect on s, in equilibrium, then i will set ψ^i_ℓ high enough (in equilibrium) to motivate these purchases.

Third, note that instead of introducing a set of profit-maximizing firms, we are simply positing that production maximizes aggregate profits subject to a single, abstract production technology. If we modeled the firms explicitly, we would have to model how their behavior responded to the diverse preferences of their owners, not all of whom here care only about profit maximization. We would likewise have to explore the possibility that some distributions of firm ownership are incompatible with equilibrium. Individuals with strong preferences regarding the production levels of some goods might, for instance, be motivated to concentrate their capital holdings in a few relevant firms so as to wield influence as "activist investors". To restrict our analysis to the equilibrium consequences of market purchases, we will therefore simply assume that production proceeds along profit-maximizing lines.

Finally, we will impose a technical condition on the $w^i(\cdot)$, as noted above. In particular, we will assume that the (positive or negative) ethical impacts of production grow subquadratically in total production levels, in the sense that

$$\lim_{a \to \infty} \frac{\nabla w^i(as)}{a} = 0 \tag{12}$$

as long as

$$s_\ell \neq 0 \quad \forall \ell. \tag{13}$$

It follows from (3) that (13) will hold in equilibrium.

This condition ensures that, when i is a small player in a large economy, her range of feasible impacts on total production is small enough that she cares negligibly about her second-order impacts on total production. That is, if the levels of the goods in existence begin at some s^0 satisfying (13), then

$$w^i(s^0 + \Delta s) \approx w^i(s^0) + \Delta s \cdot \nabla w^i(s^0) \tag{14}$$

for any equilibrium impact Δs feasible for i. Put another way, (12) ensures that as we multiply the population—holding fixed each I-sized subpopulation's distribution of endowments and utility functions—by ever larger factors k, so that productions and endowments are also multiplied by ever larger k, i's utility function around a given s^0 can be approximated ever more closely (and perfectly in the limit) by

$$u^i(x^i, s) = v^i(x^i) + w^i(s^0) + (s - s^0) \cdot \nabla w^i(s^0), \tag{15}$$

and her demand function can likewise be approximated ever more closely by the demand function implied by (1) with (15) substituted for $u^i(x^i, s)$.

Note that the ethical impacts of production grow linearly (so, subquadratically) in the size of the economy if they exhibit constant returns to scale. As in the case of production, constant returns to scale here might be justified on the grounds of a "replication argument". If we doubled the earth and everything in it, the value or disvalue present would presumably double as well (at least in, say, a total utilitarian ethical framework), along with the ethically relevant externalities of the total production of each good.

3 Results

In this context, with individuals partially internalizing the impacts of their purchases on total production levels, the relevant notion of market equilibrium is not Walrasian equilibrium, or even the "Walrasian equilibrium with slack" (WES) necessary to deal with the possibility of nonsatiation. Nevertheless, let us now define WES in this context. An alternative equilibrium concept will be defined subsequently.

Definition 1. *Given aggregate endowments ω, an aggregate demand function $x(p, \psi)$, an aggregate production function $y(p)$, and ethical weights ψ, p^* is a Walrasian equilibrium with slack (WES) if*

$$y(p^*) + \omega = x(p^*, \psi). \tag{16}$$

Let $p^*(\omega, x, y, \psi)$ denote the set of generic WESes compatible with ω, x, y, and ψ. Given ω, x, y, ψ^*, and $p \in p^*(\omega, x, y, \psi^*)$, let $p^*_{\omega,x,y,p}(\psi)$, defined locally around ψ^*, denote the WES p^* compatible with ω, x, y, and ψ that is in the neighborhood of p. Note that, since p is a generic WES, p^* exists and is unique.

Definition 2. *Given aggregate endowments ω, an aggregate demand function $x(p, \psi)$, and an aggregate production function $y(p)$, (p^*, ψ^*) is a Walrasian equilibrium with supply externalities (WESE) if p^* is a WES given $(\omega,\ x,\ y,\ \psi^*)$ and*

$$\psi^{*i}_\ell = \dots \text{ think about how to define this.} \tag{17}$$

[The issue here is that we want to define prices as a function of *demand functions*, rather than ψ, so that we can have ψ^i be the derivative of w^i with respect to demands (locally around some WES). So we really do have to go back and frame things in terms of people submitting overall demand functions, at least from some class. Maybe that class is just "all demand functions that maximize $\tilde{u}^i$ for some $\tilde{u}^i$ compatible (given some ψ^i) with (1)"... but it would be nice if we could be more general.]

Let $\mathbb{0}_L$ and $\mathbb{1}_L$ denote the L-length vectors of zeroes and ones respectively. Let $D(a)$, for an arbitrary vector a, denote the diagonal $\text{Len}(a) \times \text{Len}(a)$ matrix M with $M_{\ell\ell} = a_\ell$ for $\ell = 1, ..., \text{Len}(a)$.

Given a price vector p and ethical weight matrix ψ, define the following terms:

$$\begin{aligned}
\delta &\triangleq \text{The gradient of the aggregate Engel curve, i.e. } \nabla_\pi x(p, \psi, \pi) && (18)\\
J_s(p) &\triangleq \text{The Jacobian of } s(p)\\
J_x(p, \psi) &\triangleq \text{The Jacobian of } x(p, \psi) \text{ with respect to } p\\
f(p, \psi) &\triangleq \text{The generalized inverse } f \text{ of } J_s - J_x \text{ with } f\delta = \mathbb{0}_L \text{ and } \mathbb{1}_L \cdot f = \mathbb{0}_L\\
\sigma(p) &\triangleq \text{The matrix of cross-price elasticities of supply at } p\\
\varepsilon(p, \psi) &\triangleq \text{The matrix of cross-price elasticities of demand at } (p, \psi)\\
\phi(p, \psi) &\triangleq \text{The generalized inverse } \phi \text{ of } \sigma - \varepsilon \text{ with } \phi D(s(p))^{-1}\delta = \mathbb{0}_L \text{ and } p \cdot \phi = \mathbb{0}_L.
\end{aligned}$$

We can now provide a closed-form solution for ψ^* in any WESE. That is, if we know certain aggregate statistics—namely the gradient of the aggregate Engel curve, and either the Jacobians of supply and demand or (equivalently) the matrices of cross-price elasticities of supply and demand and the total supply levels—we can determine the ethical weight that an individual with some ethical preferences should assign to each good.

Proposition 1 (Ethical weights in WESE)**.**
Given a WESE (p^*, ψ^*),

$$\begin{aligned}
\psi^{*i} &= \left[J_s(p^*)\ f(p^*, \psi^*)\right]^T\ \nabla w^i(s(p^*)) && (19)\\
&= \left[D(s(p^*))\ \sigma(p^*)\ \phi(p^*, \psi^*)\ D(s(p^*))^{-1}\right]^T\ \nabla w^i(s(p^*)) && (20)
\end{aligned}$$

for all i.

Proof. See Appendix A.1. □

Proposition 2 (Non-satiation in WESE given additive separability)**.**
Suppose $v^i(\cdot)$ is additively separable, for all i, into increasing components all of which are weakly concave and at most one of which is not strictly concave. Then, in any WESE (p^*, ψ^*), $\psi^{*i} \not\ll 0\ \forall i$, and all individuals exhaust their budgets.

Proof. See Appendix A.2. □

Determining the precise circumstances under which a WESE exists is difficult. However, we can show existence under certain conditions.

To begin, let

$$S \triangleq \big\{ s : s = \omega + y \text{ for some } y \text{ on the frontier of } Y \text{ and } s_\ell \geq 0\ \forall \ell \big\}. \tag{21}$$

That is, let S denote the set of just-feasible supply vectors.

Since the frontier of Y is strictly concave, the marginal rates of transformation between goods are bounded throughout S. Let P denote the relative price vectors, corresponding to these marginal rates of transformation, with components summing to 1. Note that P is compact, that $p \gg 0\ \forall p \in P$, and that, as long as we are in an interior solution (with $s \in S^\circ$), the relative price vector p for which s is profit-maximizing must be within P.

Also, let

$$\bar{P}_\ell \triangleq \max_{p \in P} \frac{p_L}{p_\ell}. \tag{22}$$

Because P is compact and $p \gg 0\ \forall p \in P$, (22) is well-defined.

Finally, let e_ℓ denote the vector with a 1 in place ℓ and 0 elsewhere, and let

$$\bar{\bar{\psi}}^i_\ell \triangleq \max_{p \in P, k \leq L, m \leq L} \big| J_s(p) e_k \cdot \nabla w^i(s(p)) \big| \big| e_\ell \cdot J_s(p) e_m \big| \quad \forall \ell < L. \tag{23}$$

Because P is compact and the functions composed are continuous in p, the maximum exists.

Proposition 3 (Existence of WESE given additive separability and other restrictions)**.**
Suppose every individual i has a felicity function $v^i(\cdot)$ of the form

$$v^i(x^i) = \sum_{\ell=1}^{L-1} \tilde{v}^i_\ell(x^i_\ell) + t^i \cdot x^i, \tag{24}$$

where the coefficient of relative risk aversion of each $\tilde{v}^i_\ell$ is everywhere within $(0, 1]$. *Suppose also that, for all i and all $\ell < L$, $\tilde{v}^i_\ell$ is differentiable and satisfies the lower and upper Inada conditions, and*

$$t^i_\ell \geq \bar{\bar{\psi}}^i_\ell, \tag{25}$$

$$t^i_L > \bar{P}_\ell \Big(t^i_\ell + \bar{\bar{\psi}}^i_\ell \Big), \text{ and} \tag{26}$$

$$\omega^i_\ell > \tilde{v}^{i\prime-1}_\ell \Big(\frac{t^i_L}{\bar{P}_\ell} - t^i_\ell - \bar{\bar{\psi}}^i_\ell \Big). \tag{27}$$

Then a WESE exists.

Furthermore, in any such WESE, $\psi^i_L = 0$ for all i.

Proof. See Appendix A.3. □

Proposition 3 demonstrates that WESEs do exist under at least some conditions. We will now show that they exist under very different conditions as well. Though both sets of conditions are highly restrictive, this pair of demonstrations may suggest that they exist more widely.

Proposition 4 (Existence of WESE given symmetric utility and endowments)**.** *Suppose that $y(\cdot)$ is symmetric across goods and that, for each i, $v^i(\cdot)$ and $w^i(\cdot)$ are symmetric across goods, that $v^i(\cdot)$ (with $\psi^i = 0$) implies a strictly increasing and strictly quasiconcave demand function $x^i(\cdot)$ satisfying the gross substitutes property, and that ω^i_ℓ is the same for all ℓ. Then a WESE exists in which $\psi^i = \mathbb{0}_L$ for all i.*

Proof. See Appendix A.4. □

[Explanation of how preferences over total supply are more general than preferences over others' consumption levels in this context]

[How to approximate (63) when J_s, J_x, and/or (especially) ∇w^i are sparse; some estimates of how far off the naive approach tends to be]

4 References

Bergstrom, Theodore, Lawrence Blume, and Hal Varian, "On the Private Provision of Public Goods," *Journal of Public Economics*, 1986, *29* (1), 25–49.

Covarrubias, Enrique, "Global Invertibility of Excess Demand Functions," 2013. Banco de Mexico working paper.

Dufwenberg, Martin, Paul Heidhues, Georg Kirchsteiger, Frank Riedel, and Joel Sobel, "Other-Regarding Preferences in General Equilibrium," *Review of Economic Studies*, 2011, *78* (2), 613–39.

Ellickson, Bryan, *Competitive Equilibrium: Theory and Applications*, Cambridge, UK: Cambridge University Press, 1994.

Konovalov, Alexander, "The Core of an Economy with Satiation," *Economic Theory*, 2005, *25*, 711–719.

Kreps, David M., *A Course in Microeconomic Theory*, Princeton, NJ: Princeton University Press, 1990.

Roberts, John and Andrew Postlewaite, "The Incentives for Price-taking Behavior in Large Exchange Economies," *Econometrica*, 1976, *44* (1), 115—-27.

Sobel, Joel, "Generous Actors, Selfish Actions: Markets with Other-Regarding Preferences," *International Review of Economics*, 2009, *56*, 3–16.

Wilkinson, Hayden, "Market Harms and Market Benefits," *Philosophy & Public Affairs*, 2022. Online preprint.

Appendices

A Proofs

A.1 Proof of Proposition 1

Fix $\{u^i\}$ and $\{\omega^i\}$, and let $\bar{p}$ denote a WES compatible with some matrix of ethical weights $\bar{\psi}$. Let $\bar{x}$ denote the consumption levels implied by $\bar{p}$.

Except in certain edge cases, for all ψ close to $\bar{\psi}$ there is a unique WES close to $\bar{p}$. Starting from $(\bar{p}, \bar{\psi})$, we will now determine the ψ that individuals will "set for themselves" so as to maximize their respective utilities in the corresponding equilibrium. We will simply assume for now that all parties exhaust their budgets in any equilibrium.

Suppose i shifts her demand by Δx^i while staying within her budget constraint—i.e. maintaining

$$p \cdot \Delta x^i \leq 0. \tag{28}$$

As long as the responses of equilibrium relative prices to these demand shifts are differentiable around $(\bar{p}, \bar{\psi})$, which they must be in any generic equilibrium, we can define the (infinitesimal) equilibrium effect of any such marginal demand-change on the price of each good as a linear function of the demand-changes. In fact there are multiple such linear functions which would have the necessary effect on equilibrium *relative prices*; let us denote the set of such matrices by $\mathcal{F}(\bar{p}, \bar{\psi})$, with elements F.

Consider a demand shift that keeps i precisely on her budget constraint a Δx^i such that

$$p \cdot \Delta x^i = 0. \tag{29}$$

Since real productive capacity has not changed following i's demand shift, we must always, for any $F \in \mathcal{F}(\bar{p}, \bar{\psi})$, have

$$\sum_{\ell=1}^{L} \Delta x_\ell^i \sum_{k=1}^{L} F_{k\ell} \left[\frac{\partial s_m(\bar{p})}{\partial p_k} - \frac{\partial x_m(\bar{p}, \bar{\psi})}{\partial p_k} \right] = \Delta x_m^i \ \ \forall m. \tag{30}$$

Alternatively, we can replace the $F_{k\ell}$ in (30) by

$$\Phi_{k\ell} \triangleq F_{k\ell} \frac{s_\ell(\bar{p})}{\bar{p}_k}, \tag{31}$$

multiply the bracketed expression by

$$\frac{\bar{p}_k}{s_m(\bar{p})} \frac{s_m(\bar{p})}{s_\ell(\bar{p})}, \tag{32}$$

and factor out the $s_m(\bar{p})/s_\ell(\bar{p})$ term. Recalling that $s = x$ in equilibrium, these changes let us represent equality (30) in elasticity terms:

$$\sum_{\ell=1}^{L} \frac{\Delta x_\ell^i}{s_\ell(\bar{p})} \sum_{k=1}^{L} \Phi_{k\ell} \big(\sigma_{mk}(\bar{p}) - \varepsilon_{mk}(\bar{p}, \bar{\psi}) \big) = \frac{\Delta x_m^i}{s_m(\bar{p})} \ \ \forall m, \tag{33}$$

where σ and ε denote the matrices of cross-price elasticities of supply and (uncompensated) demand respectively.

Of course, an incremental shift in expenditure will typically, in equilibrium, marginally impact the relative prices of many goods, including other goods i buys and goods with which she is endowed. As in the conventional case of self-interested individuals, i can ignore these price impacts for budgeting purposes when I is large. However, i cannot ignore these price impacts for the sake of estimating the equilibrium impacts of her purchases on total production. As I grows (and as the corresponding economy is multiplied), $F_{k\ell}$ falls with $1/I$, but $\partial s_\ell(\bar{p})/\partial p_k$ grows with I. $\phi_{k\ell}$, $\sigma_{k\ell}$, and $\varepsilon_{k\ell}$ converge to constants.

Also, for any finite I, we must remember that the (exactly or approximately) $1/p_\ell$ units of ℓ newly acquired by i come from increases in the production of ℓ and/or decreases in *others'* consumption of ℓ. Likewise, we are here stipulating that i's consumption of all $m \neq \ell$ is fixed. The derivative of aggregate demand in (30) would therefore, in a more exact finite-population expression, be a derivative only of demand aggregated over $j \neq i$. As I grows, however, i's demand responses to price become an ever smaller fraction of aggregate demand responses to price;

$$F_{k\ell} \sum_{j \neq i} \frac{\partial x_m^j(\bar{p}, \bar{\psi})}{\partial p_k} \quad \to \quad F_{k\ell} \frac{\partial x_m(\bar{p}, \bar{\psi})}{\partial p_k} \quad \forall m. \tag{34}$$

Therefore (30) holds precisely in the limit.

Writing (31) in matrix notation, we have

$$\Phi(\bar{p}, \bar{\psi}) = F(\bar{p}, \bar{\psi}) s(\bar{p}) \big(\mathbb{1}_L \cdot D(\bar{p})^{-1}\big), \tag{35}$$

where $\mathbb{1}_L$ denotes the L-length vector of ones and $D(p)$, for an arbitrary vector p, denotes the diagonal $\text{Len}(p) \times \text{Len}(p)$ matrix with $D_{\ell\ell} = p_\ell$ for $\ell = 1, ..., \text{Len}(p)$. Therefore let $\mathcal{F}[s/p]$ denote the set of matrices Φ satisfying (31) for some $F \in \mathcal{F}(\bar{p}, \bar{\psi})$.

Writing (30) in matrix notation, we have, for any Δx^i satisfying (29) and any $F \in \mathcal{F}(\bar{p}, \bar{\psi})$,

$$\big(J_s(\bar{p}) - J_x(\bar{p}, \bar{\psi})\big) \; F \; \Delta x^i = \Delta x^i, \tag{36}$$

where J_s denotes the Jacobian of $s(p)$ (or, equivalently, the Jacobian of $y(p)$) and J_x denotes the Jacobian of $x(p, \psi, \pi)$ with respect to p. Likewise, writing (33) in matrix notation, we have, for any Δx^i satisfying (29) and any $\Phi \in \mathcal{F}[s/p](\bar{p}, \bar{\psi})$,

$$\begin{aligned} &\big(\sigma(\bar{p}) - \varepsilon(\bar{p}, \bar{\psi})\big) \; \Phi \; D(s(\bar{p}))^{-1} \Delta x^i = D(s(\bar{p}))^{-1} \Delta x^i \\ \implies \; & D(s(\bar{p}))\big(\sigma(\bar{p}) - \varepsilon(\bar{p}, \bar{\psi})\big) \; \Phi \; D(s(\bar{p}))^{-1} \Delta x^i = \Delta x^i. \end{aligned} \tag{37}$$

$J_s p = 0$. If prices all rise in proportion to their current levels, then the price vector has simply been rescaled, and supply levels will not change. $J_x p = 0$ likewise, so $(J_s - J_x)p = 0$ as well. And p spans its nullspace: by the strict concavity of $y(p)$ and $x(p, \bar{\psi}, \pi)$, any change to relative prices shifts supply and demand in opposite directions, so $(J_s - J_x)q \neq 0$ when q is not a multiple of p.[3] $J_s - J_x$ is thus singular, with

$$\text{Rank}(J_s - J_x) = L - 1. \tag{38}$$

It follows immediately from (36) that the column space of $J_s - J_x$ contains all Δx^i satisfying (29). This space of Δx^i is of dimension $L - 1$. It then follows from

[3] I guess this is what changes when the equilibrium $\bar{p}$ is a "multiple root"? In this case, the equilibrium impacts of marginal demand changes on prices are indeed undefined: slight changes to demands can locally eliminate equilibria, or generate multiple local equilibria where there had been only one. But this shouldn't ever be a problem, since our assumptions imply that the WES for any given ψ will be unique.

(38) that the column space of $J_s - J_x$ *is* the set of Δx^i satisfying (29). F is thus a generalized inverse of $J_s - J_x$.

It follows by reasoning analogous to that two paragraphs above that $\mathbb{1}_L$ spans the nullspace of $\sigma - \varepsilon$, and that, since $D(s)$ is invertible,

$$\text{Rank}(\sigma - \varepsilon) = \text{Rank}(D(s)(\sigma - \varepsilon)) = L - 1. \tag{39}$$

By reasoning analogous to that of the previous paragraph, the column space of $D(s)(\sigma - \varepsilon)$ is thus the set of Δx^i satisfying (29). It then follows from (37) that $\Phi D(s)^{-1}$ is a generalized inverse of $D(s)(\sigma - \varepsilon)$, and thus that Φ is a generalized inverse of $\sigma - \varepsilon$.

Let

$$\delta^i \triangleq \nabla_\pi x^i(p, \psi, \pi) \text{ and} \tag{40}$$

$$\delta \triangleq \sum_i \delta^i = \nabla_\pi x(p, \psi, \pi). \tag{41}$$

Observe that $\delta \neq 0$. Letting I denote a pure consumer, and recalling our assumption that at least one exists, we have $p \cdot \delta^I > 0$. We have $p \cdot \delta^i \geq 0$ $\forall i \neq I$ as well, with equality holding iff i is satiated (as may happen with $\psi^i \ll 0$). Thus, summing across i, $p \cdot \delta > 0$.

As a result, if some i adjusts her demand by $\Delta x^i = -\delta$, profits π can simply increase until aggregate demand equals supply. There will be no change to relative prices. We must thus have $F\delta$ proportional to p for all $F \in \mathcal{F}$.

Let us now construct the unique $f \in \mathcal{F}$ for which the sum of price changes resulting from a demand shift always equals zero: in this case, for example, imposing

$$f\delta = \mathbb{0}_L. \tag{42}$$

Let UNV^T denote a singular value decomposition of $J_s - J_x$ with $N_{\ell\ell} \neq 0$ for $\ell < L$ and $N_{LL} = 0$. Because f is a generalized inverse of $J_s - J_x$, we must by (38) have

$$f = V \begin{bmatrix} N_1^{-1} & A \\ B & C \end{bmatrix} U^T, \tag{43}$$

where N_1 is the $(L-1) \times (L-1)$ principal submatrix of N and A, B, and C are $(L-1) \times 1$, $1 \times (L-1)$, and 1×1 respectively.

Since V is invertible, (42) reduces to

$$\begin{bmatrix} N_1^{-1} & A \\ B & C \end{bmatrix} \tilde{\delta} = \mathbb{0}_L \tag{44}$$

$$\implies A_\ell = -\frac{\tilde{\delta}_\ell}{\tilde{\delta}_L} \frac{1}{N_{\ell\ell}}, \quad \ell = 1, ..., L-1, \text{ and} \tag{45}$$

$$C = -\frac{1}{\tilde{\delta}_L} \sum_{\ell=1}^{L-1} B_\ell \tilde{\delta}_\ell, \tag{46}$$

where $\tilde{\delta} \triangleq U^T \delta$.

Furthermore, since the sum of normalized price changes resulting (by f) from any demand change equals zero, we have

$$\mathbb{1}_L \cdot f = \mathbb{0}_L^T. \tag{47}$$

Since U^T is invertible, (47) reduces to

$$\tilde{V} \begin{bmatrix} N_1^{-1} & A \\ B & C \end{bmatrix} = \mathbb{0}_L^T \tag{48}$$

$$\implies B_\ell = -\frac{\tilde{V}_\ell}{\tilde{V}_L} \frac{1}{N_{\ell\ell}}, \quad \ell = 1, ..., L-1, \tag{49}$$

where $\tilde{V} \triangleq \mathbb{1}_L \cdot V$. By (46), this also gives us C. f is thus fully constructed.

Likewise, we now know not only that Φ is a generalized inverse of $\sigma - \varepsilon$, but also that $\Phi D(s)^{-1}\delta$ is proportional to p, for all $\Phi \in \mathcal{F}[s/p]$. Let us now construct the unique $\phi \in \mathcal{F}[s/p]$ for which the sum of price changes resulting from a *proportional* demand shift always equals zero: in this case, for example, imposing

$$\phi D(s)^{-1}\delta = \mathbb{0}_L. \tag{50}$$

Let $\underline{UNV}^T$ denote a singular value decomposition of $\sigma - \varepsilon$ with $\underline{N}_{\ell\ell} \neq 0$ for $\ell < L$ and $\underline{N}_{LL} = 0$. Because ϕ is a generalized inverse of $\sigma - \varepsilon$, we must by (39) have

$$\phi = \underline{V} \begin{bmatrix} \underline{N}_1^{-1} & \underline{A} \\ \underline{B} & \underline{C} \end{bmatrix} \underline{U}^T, \tag{51}$$

where $\underline{N}_1$ is the $(L-1) \times (L-1)$ principal submatrix of $\underline{N}$ and $\underline{A}$, $\underline{B}$, and $\underline{C}$ are $(L-1) \times 1$, $1 \times (L-1)$, and 1×1 respectively.

Since $\underline{V}$ is invertible, (50) reduces to

$$\begin{bmatrix} \underline{N}_1^{-1} & \underline{A} \\ \underline{B} & \underline{C} \end{bmatrix} \underline{\tilde{\delta}} = \mathbb{0}_L \tag{52}$$

$$\implies \underline{A}_\ell = -\frac{\underline{\tilde{\delta}}_\ell}{\underline{\tilde{\delta}}_L} \frac{1}{\underline{N}_{\ell\ell}}, \quad \ell = 1, ..., L-1, \text{ and} \tag{53}$$

$$\underline{C} = -\frac{1}{\underline{\tilde{\delta}}_L} \sum_{\ell=1}^{L-1} \underline{B}_\ell \underline{\tilde{\delta}}_\ell, \tag{54}$$

where $\underline{\tilde{\delta}} \triangleq \underline{U}^T D(s)^{-1}\delta$.

Furthermore, the sum of normalized price changes resulting (by ϕ) from any demand change—i.e. the dot product of the price vector and the vector of proportional price changes induced by ϕ—equals zero. We thus have

$$\bar{p} \cdot \phi = \mathbb{0}_L^T. \tag{55}$$

Since $\underline{U}^T$ is invertible, (55) reduces to

$$\underline{\tilde{V}} \begin{bmatrix} \underline{N}_1^{-1} & \underline{A} \\ \underline{B} & \underline{C} \end{bmatrix} = \mathbb{0}_L^T \tag{56}$$

$$\implies \underline{B}_\ell = -\frac{\underline{\tilde{V}}_\ell}{\underline{\tilde{V}}_L} \frac{1}{\underline{N}_{\ell\ell}}, \quad \ell = 1, ..., L-1, \tag{57}$$

where $\underline{\tilde{V}} \triangleq p \cdot \underline{V}$. By (54), this also gives us $\underline{C}$. ϕ is thus fully constructed.

Having constructed f (or ϕ), we can now characterize i's impact on total production resulting from a given feasible deviation in demand from $x^i(\bar{p}, \bar{\psi})$ to x^i. In particular, the vector of supply differences is given by

$$\begin{aligned} \Delta s &= J_s(\bar{p})\ f(\bar{p}, \bar{\psi})\ (x^i - x^i(\bar{p}, \bar{\psi})) &(58)\\ &= D(s(\bar{p}))\ \sigma(\bar{p})\ \phi(\bar{p}, \bar{\psi})\ D(s(\bar{p}))^{-1}(x^i - x^i(\bar{p}, \bar{\psi})). &(59) \end{aligned}$$

Substituting (58) for $s - s^0$ and $s(\bar{p})$ for s^0 into (15), we have

$$\begin{aligned} u^i(x^i) &= v^i(x^i) + w^i(s(\bar{p})) + \left[J_s(\bar{p})\ f(\bar{p}, \bar{\psi})\ (x^i - x^i(\bar{p}, \bar{\psi}))\right] \cdot \nabla w^i(s(\bar{p})) &(60)\\ &= v^i(x^i) + w^i(s(\bar{p})) \\ &\quad + \left[D(s(\bar{p}))\ \sigma(\bar{p})\ \phi(\bar{p}, \bar{\psi})\ D(s(\bar{p}))^{-1}\ (x^i - x^i(\bar{p}, \bar{\psi}))\right] \cdot \nabla w^i(s(\bar{p})) &(61) \end{aligned}$$

where the s argument has disappeared from $u^i(\cdot)$ because supply is now expressed as a function of x^i.

Noting the vector of "ethical" coefficients on x^i (i.e. the coefficients on the terms outside $v^i(x^i)$), i's objective function reduces to

$$\tilde{u}^i(x^i, \psi^i) = v^i(x^i) + \psi^i \cdot x^i, \tag{62}$$

where

$$\psi^i = \left[J_s(\bar{p})\ f(\bar{p}, \bar{\psi})\right]^T \ \nabla w^i(s(\bar{p})) \tag{63}$$

$$= \left[D(s(\bar{p}))\ \sigma(\bar{p})\ \phi(\bar{p}, \bar{\psi})\ D(s(\bar{p}))^{-1}\right]^T \ \nabla w^i(s(\bar{p})). \tag{64}$$

A.2 Proof of Proposition 2

Let us begin from an equilibrium and corresponding ethical weight matrix $(\bar{p}, \bar{\psi})$. Holding prices fixed, let us consider the impact of a marginal increase in dividends on the demands of some individual i who is nonsatiated around $(\bar{p}, \bar{\psi}, \pi(\bar{p}, \bar{\psi}))$.

Suppose that there is a good m in which $v^i(\cdot)$ is not strictly concave around $x^i_m(\bar{p}, \bar{\psi}, \pi(\bar{p}, \bar{\psi}))$ and that $x^i_m(\bar{p}, \bar{\psi}, \pi(\bar{p}, \bar{\psi})) > 0$. Then a marginal increase in dividends increases i's income by θ^i, and all of this income is spent on good m. That is, $\delta^i_m = \theta^i/p_m$, and $\delta^i_\ell = 0$ for $\ell \neq m$.

[THIS ISN'T DONE—I need to deal with the case in which $x^i_m = 0$. I also need to deal with the edge case where i is indifferent about consuming m but satiated in all other goods.] [The bit below isn't quite done either...]

Suppose that, around $x^i(\bar{p}, \bar{\psi}, \pi(\bar{p}, \bar{\psi}))$, $v^i(\cdot)$ is strictly concave in all goods k for which $x^i_k > 0$.

Because j's marginal rates of substitution across goods stay constant, we know that j adjusts his spending by some $\delta^j(\bar{p}, \bar{\psi})$ such that his marginal quasi-utility in each good, $\partial\tilde{u}^j/\partial x^j_\ell$, falls by the same proportion for all ℓ. By the assumption of no inferior goods, In particular, since j's marginal quasi-utility in each good must have been proportional to its price, we must have

$$H^j(\bar{p}, \bar{\psi})\delta^j(\bar{p}, \bar{\psi}) = \alpha\bar{p} \tag{65}$$

for some $\alpha > 0$, where H^j denotes the Hessian of $\tilde{u}^j$ with respect to x^j.

By the strict concavity of $v^j(x^j)$ and the weak concavity of $\psi^j \cdot x^j$ with respect to x^j, and the fact that the sum of concave functions (with at least one strictly concave) is strictly concave, $\tilde{u}^j(\cdot)$ is strictly concave with respect to x^j. Thus H^j is everywhere negative definite, and thus invertible. So we have

$$\delta^j(\bar{p}, \bar{\psi}) = \alpha\left[H^j(\bar{p}, \bar{\psi})\right]^{-1}\bar{p}. \tag{66}$$

It also follows that $\delta^j \gg 0$ everywhere.

We must have α such that the marginal cost of this demand change equals the marginal increase in j's budget:

$$\alpha\bar{p}\cdot\left[H^j(\bar{p},\bar{\psi})\right]^{-1}\bar{p}=1 \tag{67}$$

$$\implies \alpha=\frac{1}{\bar{p}\cdot\left[H^j(\bar{p},\bar{\psi})\right]^{-1}\bar{p}} \tag{68}$$

$$\implies \delta^j(\bar{p},\bar{\psi})=\frac{1}{\bar{p}\cdot\left[H^j(\bar{p},\bar{\psi})\right]^{-1}\bar{p}}\left[H^j(\bar{p},\bar{\psi})\right]^{-1}\bar{p}. \tag{69}$$

Now suppose that, for some i, $x^j(\dot{)}$ is locally nonsatiated around $(\bar{p},\bar{\psi},\pi(\bar{p},\bar{\psi}))$ for all $j\neq i$. Suppose also that, starting from a demand x^i that exhausts i's budget at $(\bar{p},\pi(\bar{p},\bar{\psi}))$, i chooses $\Delta x^i\ll 0$ equal to the negative of the share-weighted average of individuals' δ^j vectors:

$$\Delta x^i=-\delta,\ \text{where}\ \delta\triangleq\sum_j\theta^j\delta^j(\bar{p},\bar{\psi})\approx\sum_{j\neq i}\theta^j\delta^j(\bar{p},\bar{\psi}) \tag{70}$$

with the approximation holding exactly as I grows "large".

This shift in demand by i increases profits (by $-p\cdot\Delta x^i=1$), and this increase in profits increases others' collective demands by $-\Delta x^i$. With aggregate demands left unchanged, relative prices and supply levels remain constant as well. It follows that the ethical impact of Δx^i, from i's perspective, equals zero—i.e. that

$$\psi^i\cdot\Delta x^i=0 \tag{71}$$

—and thus that $\psi^i_\ell\geq 0$ for at least one ℓ, and thus that i is locally nonsatiated. In other words, given an equilibrium $(\bar{p},\bar{\psi})$ in which all $j\neq i$ are nonsatiated, the best response ψ^i implies that i is also nonsatiated.

A.3 Proof of Proposition 3

Suppose all individuals i have quasi-utility functions $\tilde{u}^i$ that take the form

$$\tilde{u}^i(x^i)=v^i(x^i)+\bar{\psi}^i\cdot x^i, \tag{72}$$

where felicity functions v^i take the form

$$v^i(x^i)=\sum_{\ell=1}^{L-1}\tilde{v}^i_\ell(x^i_\ell)+t^i\cdot x^i, \tag{73}$$

where the coefficient of relative risk aversion of each $\tilde{v}^i_\ell$ is everywhere within $(0,1]$ and $t^i+\bar{\psi}^i\geq 0$. Suppose also that $\bar{\psi}^i_L=0$, that the $\tilde{v}^i_\ell$ are all differentiable and satisfy the lower and upper Inada conditions, and that

$$t^i_L>\bar{P}_\ell\big(t^i_\ell+\bar{\psi}^i_\ell\big)\ \ \forall\ell<L, \tag{74}$$

so that, for large enough values of x^i_ℓ, i prefers purchases of L to further purchases of ℓ.

Individuals' demand functions $x^i(p, \bar{\psi}, \pi)$, written as functions of prices and profits holding ethical weights fixed, are as if derived from utility functions of the above description. Note that the conditions above guarantee that each $x^i(\cdot)$ satisfies the weak gross substitutes property (GS) and that individuals exhaust their budgets.

Finally, suppose that endowments ω satisfy

$$\begin{aligned} & t^i_L > \bar{P}_\ell\big(\tilde{v}^{i\prime}_\ell(\omega^i_\ell) + t^i_\ell + \bar{\psi}^i_\ell\big) \;\; \forall \ell < L \\ \iff & \omega^i_\ell > \tilde{v}^{i\prime -1}_\ell\Big(\frac{t^i_L}{\bar{P}_\ell} - t^i_\ell - \bar{\psi}^i_\ell\Big), \end{aligned} \tag{75}$$

so that, in effect, inequality (74) holds not only in the limit as $x^i_\ell \to \infty$ but also at the endowment point. This ensures that $x^i_L(p, \bar{\psi}, \pi) > 0 \; \forall p \in P$ and all π. For any feasible interior price vector, consuming the endowment would leave the marginal utility of purchasing L higher than the marginal utility of purchasing more of *any* other good, so some L must be purchased, even if $\omega^i_L = 0$ and $\pi = 0$. And of course, if $\omega^i_L > 0$ or i receives dividends, x^i_L must be higher than it would be otherwise.

We thus have a (unique) WES $\bar{p} \in P$, with $x^i_\ell(\bar{p}, \bar{\psi}) > 0$ for all i and ℓ. (The inequality holds for all $\ell < L$ by the lower Inada condition, and for L by the reasoning above.) [Something about how we're thus at an interior supply vector for any $p \in P$, so everything checks out so far.]

Given $\bar{p}$, $\bar{\psi}$, $s(\cdot)$ and $x(\cdot)$, define δ and f as in (18). Note that, because marginal consumption all goes to good L given $x^i_L > 0 \; \forall i$, here $\delta = e_L/\bar{p}_L$.

The vector of price-impacts induced by individual i's demand-shift $\Delta x^i = e_\ell$ for some $\ell < L$, starting from WES $\bar{p}$, is the same as the vector of price-impacts induced by

$$\Delta x^i = b_\ell \triangleq e_\ell - \bar{p}_\ell \delta, \tag{76}$$

since $f\delta = 0$.

We know that $\bar{p} \cdot b_\ell = 0$: purchasing a unit of good ℓ costs $\bar{p}_\ell$, and purchasing the basket of goods denoted δ costs one unit, since it is defined to be the increase in aggregate demand resulting from a unit increase in profits (and here, because $\tilde{u}^i$ is strictly increasing in the purchase of at least one good (and indeed of all goods), individuals exhaust their budgets). Therefore, by the reasoning following (38), b_ℓ is in the column space of $J_s(\bar{p}) - J_x(\bar{p})$.

The price-impact of b_ℓ is $fb_\ell = fe_\ell$: the ℓ^{th} column of $f(\bar{p}, \bar{\psi})$. As we will now show, we can place an upper bound on the absolute ethical impacts, for i, of this price-impact. We will first place bounds on the ethical bounds of this price-impact for any given $\bar{p}$ (but ensuring that these bounds do not depend on $\bar{\psi}$); then we will place bounds that hold across all $p \in P$.

With $\bar{p}$ and $\bar{\psi}$ still given, let

$$|f_\ell| \triangleq \sum_{k=1}^{L} |f_{k,\ell}|, \tag{77}$$

and consider the rescaled demand-shift $b_\ell/|f_\ell|$. The price-impact of this rescaled demand-shift equals $fe_\ell/|f_\ell|$. This is a vector the absolute values of whose components, by construction, sum to 1. The absolute value of each component must therefore be no greater than 1. Therefore, because ethical impacts are additive in price shifts, the absolute ethical impact, for i, of price-impact $fe_\ell/|f_\ell|$ cannot be greater than the maximum ethical impact that could result from a price-shift of the form e_k or $-e_k$, for some k:

$$\max_{k\le L} \big|J_s e_k \cdot \nabla w^i\big|. \tag{78}$$

To find a lower bound for (77), consider the absolute values of the ℓ^{th} row of $J_s - J_x$. By the assumption that the upper boundary of Y is strictly concave, J_s is positive on the diagonal and negative elsewhere. Because $x(\cdot)$ satisfies weak GS, J_x is nonpositive on the diagonal and nonnegative elsewhere. Thus the largest absolute value of $e_\ell \cdot (J_s - J_x)$ (the ℓ^{th} row of $J_s - J_x$) is no greater than the largest absolute value of the $e_\ell \cdot J_s$. Let us say that this largest absolute value appears in column m, so that it equals $|e_\ell \cdot J_s e_m|$. Since J_s exhibits no zeroes, $|e_\ell \cdot J_s e_m| > 0$.

Because $(J_s - J_x)fe_\ell = b_\ell$, the dot product of $e_\ell \cdot (J_s - J_x)$ and fe_ℓ equals $b_{\ell\ell}$, which in turn equals 1 (since $\delta = e_L/p_L$, so $\delta_\ell = 0$). Term (77) thus cannot be smaller than the value it obtains when $|f_{m,\ell}| = |e_\ell \cdot J_s e_m|^{-1}$ and $f_{k,\ell} = 0$ for $k \ne m$: namely, $|e_\ell \cdot J_s e_m|^{-1}$.

The absolute ethical impact for i of demand-shift b_ℓ ($\ell < L$) thus, across *all* feasible prices, can never exceed

$$\bar{\bar{\psi}}^i_\ell \triangleq \max_{p\in P, k\le L, m\le L} \big|J_s(p)e_k \cdot \nabla w^i(s(p))\big|\big|e_\ell \cdot J_s(p)e_m\big| \quad (\ell < L). \tag{79}$$

Because P is compact and the functions composed are continuous in p, the maximum exists. This then bounds the absolute value of the best-response ψ^i_ℓ, given $p \in P$, across all demand functions and endowments consistent with quasi-utility functions satisfying (72)–(74) and corresponding endowments satisfying (75).

Finally, let $\bar{\bar{\psi}}^i_L = 0$.

Let

$$\Psi \triangleq \Big\{\psi : \psi^i_\ell \in \Big[-\bar{\bar{\psi}}^i_\ell, \bar{\bar{\psi}}^i_\ell\Big]\Big\}. \tag{80}$$

The bounds of Ψ, as derived above, do not depend on the values of t or ψ originally chosen, as long as they satisfy (72)–(74). Therefore, suppose we impose the further condition on preferences that

$$t^i_\ell \geq \bar{\bar{\psi}}^i_\ell \quad \forall i, \ \forall \ell < L, \tag{81}$$

and, having chosen all such t^i_ℓ, maintain the corresponding condition on t^i_L from (74), namely that

$$t^i_L > \bar{P}_\ell \Big(t^i_\ell + \bar{\bar{\psi}}^i_\ell \Big) \ \ \forall i, \ \forall \ell < L. \tag{82}$$

Suppose finally that we restrict ω^i so that (75) holds for all $\psi \in \Psi$, by imposing

$$\omega^i_\ell > \tilde{v}^{i\prime -1}_\ell \Big(\frac{t^i_L}{\bar{P}_\ell} - t^i_\ell - \bar{\bar{\psi}}^i_\ell \Big) \ \ \forall i, \ \forall \ell < L. \tag{83}$$

Let $\psi(\bar{p}, \bar{\psi})$ denote the matrix of best-response ethical weights to $\bar{p}$, $\bar{\psi}$, as given by Proposition 1: i.e.

$$\psi^i(\bar{p}, \bar{\psi}) = [J_s(\bar{p}) f(\bar{p}, \bar{\psi})]^T \nabla w^i(s(\bar{p})) \quad \forall i. \tag{84}$$

Also, let $p(\bar{\psi})$ denote the (unique) WES compatible with ethical weights $\bar{\psi}$. Assumptions (81)–(83) guarantee that, for all i,

$$\begin{aligned} t^i + \psi^i(p(\bar{\psi}), \bar{\psi}) &\geq 0, \\ \psi^i_L(p(\bar{\psi}), \bar{\psi}) &= 0 \qquad \forall \bar{\psi} \in \Psi, \end{aligned} \tag{85}$$

with the latter equality holding because, as long as $x^i_L > 0 \ \forall i$, marginal purchases for all individuals go entirely to good L, so a choice to consume less of good L in isolation simply increases profits, all of which are spent on L, without changing any prices or the quantities of any goods supplied. By the reasoning up to (79), therefore, the range of $\psi(\cdot)$, over domain $P \times \Psi$, is contained in Ψ.

$\psi(\bar{p}, \bar{\psi})$ is continuous in $\bar{p}$ and $\bar{\psi}$ over $P \times \Psi$. This follows from the fact that $s(\cdot)$ and $w^i(\cdot)$ are continuously differentiable by assumption, and from the fact that $f(\bar{p}, \bar{\psi})$ is the inverse of $J_s(\bar{p}) - J_x(\bar{p}, \bar{\psi})$ (over a restricted domain), which is continuous in $\bar{p}$ and $\bar{\psi}$ over $P \times \Psi$.

$p(\bar{\psi})$ is continuous in $\bar{\psi}$. This follows from the fact that equilibrium price changes are continuous in demand changes around a generic equilibrium (the function from the latter to the former being the matrix f) and the fact that demand changes are continuous in changes to ψ [but show this more rigorously]. Also, since our assumptions ensure that $s \in S^\circ$ in equilibrium given $\bar{\psi} \in \Psi$, the range of $p(\bar{\psi})$, over domain Ψ, is contained in P.

We thus have a continuous function $(p, \psi)(\bar{p}, \bar{\psi})$ from a compact, convex set $P \times \Psi$ to itself. By Brouwer's fixed-point theorem, a fixed point exists. By definition, it is a WESE.

A.4 Proof of Proposition 4

Suppose $\psi^i = \mathbb{0}_L\ \forall i$. Then $\tilde{u}^i(\cdot) = v^i(\cdot)\ \forall i$, so by the assumptions on the $\{v^i(\cdot)\}$ there is a unique WES p^*.

We must have p^*_ℓ the same for all ℓ (and, for all i, have $x^i_\ell(p^*)$ (and thus $s^i_\ell(p^*)$) the same for all ℓ). If we did not, any permutation p' of p^* would also be a WES, with supply and demand levels permuted accordingly, by the symmetries of supply functions, demand functions, and endowments. But the WES p^* is unique.

Likewise, the best-response ethical weights $\psi(p^*, 0)$ must be equal, for each i, across ℓ.

Starting from $x^i(p^*)$, consider a demand-shift Δx^i in the direction of $[-1, -1, ..., -1]^T$. This shift will increase profits, which will be spent by others on all goods equally, returning supply levels precisely to $s(p^*)$. The ethical impact, for i, of Δx^i is thus zero. Since the ethical impact for i is also proportional to $\sum_{\ell=1}^{L} \psi^i_\ell$, and since ψ^i_ℓ is the same across ℓ, we must have $\psi^i = \mathbb{0}_L$.

Therefore $(p^*, 0)$ is a WESE.