Patience and Philanthropy

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This is a REWRITE IN PROGRESS of what I formerly called "Discounting for Patient Philanthropists".

Abstract

Philanthropists must decide to what extent to spend their resources on present philanthropic projects and to what extent to invest them for use on future philanthropic projects. Furthermore, when a philanthropist aims to provide public goods for which he is not the only funder—and, in particular, when he and the other funders have different rates of pure time preference—he must consider the ways in which his spending schedule affects that of the good's other funders. I investigate some features of the resulting dynamic public good contribution problem. Finally, I explore how the model interacts with existing academic and philanthropic discussion about the optimal timing of philanthropic spending. I conclude that standard assumptions imply that patient philanthropists should invest all or almost all their resources in most circumstances, and that the patient payoff gains from doing so are large.

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1 Introduction

Agents must allocate their resources between spending and investment for future spending. Individuals, for instance, must decide how much of their income to consume and how much of it to save for retirement. Their decision will depend in part on whether they discount the utility from their future consumption, and if so, by how much. If individuals are assumed to discount this utility exponentially, the discount rate is typically estimated to be about 2% per year. Following this revealed time preference, it is standard practice for policymakers to discount future costs and benefits likewise (see e.g. US Council of Economic Advisers (2017)).

Some potential public projects require up-front costs for long-term benefits. Climate change mitigation, broadly speaking, is widely understood to be such a project. If a pure time preference rate anywhere close to 2% per year is used in setting climate policy, therefore, we may conclude that it is optimal to pursue only mild climate change mitigation efforts: a low carbon tax, for instance. Considering it unethical to let the impatience of the present generation impose such costs on future generations, Stern (2007) famously argues that (at least) climate policy should maintain "intergenerational equity": that is, it should not reflect pure time preference. In place of a pure time discount rate of 2% per year, he proposes using a rate of 0.1% per year, intended to reflect only the annual risk of an exogenous catastrophe that renders climate change abatement efforts worthless. Stern's recommendation, in effect, is to abandon time preference per se, but to act on the supposition that the life expectancy of our civilization is about one thousand years.

This recommendation sparked a substantial literature on optimal policy by policymakers with lower rates of (quasi-)pure time preference than their constituents in general (typically 0.1%, following Stern), not just in the context of climate change. Optimal taxation problems, for example, are explored by Farhi and Werning (2007, 2010) in an overlapping generations context where individuals save insufficiently (from the patient social planner's perspective) for their descendants, and by von Below (2012), Belfiori (2017), and Barrage (2018) in the context where present production confers both future costs and future benefits (from climate damage and capital accumulation respectively). An important lesson from this literature is that patient policymakers might like to invest for future spending, but that to avoid crowding out private investment, it is optimal for them instead to subsidize private investment and tax private consumption.

Not everyone agrees with the Stern recommendation. Many, such as Nordhaus (2007), defend the standard practice on the grounds that it is undemocratic for policymakers to use lower discount rates in setting public policy than individuals use in making their own intertemporal tradeoffs. Still, thirty-eight percent of economists who study discounting, according to a recent survey (Drupp et al. (2018)), and most moral philosophers (Broome (1994)), agree that, at least when setting policy, we ought to be patient.

Many philanthropists—that is, private actors seeking to contribute to the funding of public goods—endorse patience as well. Perhaps most notably, the Effective Altruism (EA) movement brands itself in part as a community of impartial, and by extension patient, philanthropists. Furthermore, the case for patience in a philanthropic context is presumably stronger than in a political one, since philanthropists are private actors pursuing (at least sometimes) ethical goals, and are not as beholden to potentially impatient constituents. Nevertheless, there appears to be essentially no academic discussion of the implications of patience for philanthropy.

This is particularly unfortunate because of the potentially large gap between the spending behavior of today's patient philanthropists and patient-optimal philanthropic spending behavior. As noted above, according to standard economic assumptions, the patient should typically spend their resources more slowly than the impatient. A very low spending rate, however, would be a substantial departure from current common practice, even among philanthropists with expressed moral commitments to intergenerational equity. The EA community's wealthiest members, for example, are currently Dustin Moskovitz and Cari Tuna, who plan to give away almost all of their roughly \$11B net worth during their lifetimes (Matthews (2018)). Members of Giving What We Can, an EA-affiliated nonprofit that encourages individuals to pledge at least ten percent of their lifetime incomes to effective charities, have also collectively pledged an estimated \$1.5 billion, mostly from smaller donors. The Giving What We Can pledge does not impose any particular spending schedule, but the organization recommends that its members give sooner (Giving What We Can (2013)). This tendency is not unanimous: a few have argued that investment returns exceed the rate at which doing good is growing more costly (see e.g. Hanson (2013)), and some appear recently to be rethinking their old positions for giving now (see e.g. Vivalt (2019)). But advocating for less philanthropic spending and more long-term philanthropic investment is still far from mainstream in philanthropic circles, including the

EA movement, and little effort has been put into its implementation.

Perhaps ironically, most large philanthropists outside the EA movement, who typically have made no explicit commitment to patience, already spend as slowly as the law permits foundations to spend—in the United States, that is 5% of their assets per year—and this behavior has historically met with criticism from those involved in EA (see e.g. Karnofsky (2007)). Most small donors outside EA, however, also appear to spend their charity budgets as soon as they earn them. This is difficult to measure precisely, but note that individual giving by Americans totaled $427.71B in 2018 (Giving USA (2019)), whereas contributions by Americans to donor-advised funds—tax-exempt vehicles in which donors without private foundations can invest funds for future giving—totaled $37.12B in 2018 (National Philanthropic Trust (2019)).

This paper is only a first step toward a thorough and general model of patient philanthropy, but its conclusions are straightforward. Philanthropists face interesting strategic considerations that render their discounting problem different from that of typical policymakers and households. Nevertheless, they do not appear to be exceptions to the rule that the patient should spend slowly. The case for currently investing most or all "patient philanthropic resources" is therefore substantially stronger than it would seem from existing discussion. This conclusion would hold even if patient philanthropists were the only funders of the goods they intended to provide, and the presence of impatient funders further strengthens the case that patient philanthropists should invest. Finally, if the arguments presented here are broadly valid, the benefits to correct reasoning about discounting are large: patient philanthropists will often achieve their goals dramatically more effectively by spending patient-optimally than by spending impatient-optimally or naively.

In short, if it can be demonstrated that patient philanthropists should spend more slowly, these conclusions could be highly relevant to the spending of at least tens of billions of dollars. An attempt to model the philanthropist's intertemporal spending problem therefore appears to be worthwhile.

The structure of the paper is as follows.

In §2, I observe that the public good provision timing problem faced by a single funder, including a philanthropist with an unusually low discount rate, is analogous to a household's basic consumption-smoothing problem. I also estimate the importance of the discount-rate-setting problem, from a patient perspective.

In §3, I consider a setting in which a public good has multiple funders with differing discount rates, so that patient philanthropists funding this good must worry about the extent to which their saving crowds out others' saving. This work resembles existing work on private contributions to public goods (e.g. by Bergstrom et al. (1986)), in which contributors must worry about the extent to which their contributions crowd out others'. I address inter- rather than intra-temporal crowding out, however, and so assume that the net present value of the budget each party has allocated to the public good is fixed.

In §4, I consider a setting in which patient philanthropists can match contributions by their less patient counterparts, and determine the matching schedule patient philanthropists should offer. This work closely resembles existing work on optimal taxation by policymakers with lower discount rates than their constituents. As discussed above, the relevant lesson from this literature is that patient policymakers will subidize private investment and tax private consumption. Since I am addressing a patient philanthropist rather than a patient policymaker, however, I focus on the constraint that only subsidization, and not taxation, is feasible.

§5 explores the relationship between the conclusions from the earlier sections and existing discussion about the optimal timing of philanthropic spending, as produced by academics (primarily Andreoni (2018)) and by philanthropists outside academia (primarily those within the EA movement).

§6 concludes.

2 Basic model

2.1 Model

Let us begin with a model in which an agent is the sole provider of some good over an infinite horizon. Consider, for example, the case of a philanthropist providing for a penniless (but potentially long-lived) individual or lineage.

Let us denote the size of the agent's budget at time $t = 0$ by B. At each moment t, we will assume that the flow utility u achieved by providing the good is an isoelastic function, with inverse elasticity of intertemporal substitution $\eta > 0$, of the rate $x \geq 0$ at which the agent spends. That is,

$$u(x(t)) = \begin{cases} \frac{x(t)^{1-\eta}-1}{1-\eta}, & \eta \neq 1; \\ \ln(x(t)), & \eta = 1. \end{cases} \tag{1}$$

The agent faces a constant instantaneous real interest rate r and a constant instantaneous time preference rate δ. The latter might represent pure time preference, plus the risk of a catastrophe that brings the agent's utility to zero forever after. (This could be the rate of "existential catastrophe", i.e. the risk per unit time that the world ends or human civilization collapses. Less dramatically, in the case of a philanthropist that cares only about a beneficiary individual or lineage, it could represent this beneficiary's mortality risk.) There does not appear to be a standard term for the quantity we are denoting δ, but we will call it the "time preference rate", reserving the term "discount rate" for the discounting of marginal spending and the term "pure time preference rate" for time preference in a risk-free environment. We need not assume that r or δ is positive.

The agent's problem is then to choose the schedule of spending rates $x(t)$ that maximizes

$$U = \int_0^\infty e^{-\delta t} u(x(t)) dt \tag{2}$$

subject to the constraint

$$\int_0^\infty e^{-rt} x(t) dt \leq B. \tag{3}$$

Proposition 1. ***Optimal individual spending schedule***
Suppose an agent has isoelastic utility in spending parameterized by η, a constant time preference rate δ, and a budget B, and suppose she can invest her resources at a constant interest rate r. Then the agent maximizes discounted utility by following spending schedule

$$x(t) = B\, \frac{r\eta - r + \delta}{\eta}\, e^{\frac{r-\delta}{\eta} t}.$$

Proof. See Appendix A.1. □

2.2 Discussion

The above model is motivated in this paper by the scenario in which a philanthropist is the sole provider of some good. So far, it is equivalent to an infinite-horizon consumption-smoothing model under certainty, assuming either (a) no future outside income or (b) complete capital markets. (Note that the assumption of complete capital markets renders this problem the same as the problem one faces with no outside income. Given certainty and complete markets, someone with future income can borrow against her entire income stream, and B can represent current assets plus the present value of future income.) Nevertheless, given the centrality of the underlying relationship described above to the analysis of patient philanthropy below, let us now take a moment to note three of its relevant features.

First, and most importantly: In the context of a simple consumption-smoothing model, the optimal spending rate is highly sensitive to the discount rate. The patient, that is, should spend slowly. As we can see from (8) at $t = 0$, it is always optimal to spend at proportional rate $\frac{r\eta - r + \delta}{\eta}$. In particular, if $\eta = 1$, the spending rate should equal δ. For instance, philanthropists who are funding idiosyncratic projects with no other present or future funders, who discount future impacts at 0.1% per year, and who are confident that the world (or their philanthropic projects) will not soon be brought to an end, should spend only 0.1% of their budgets per year.

Second: Whether outflows are increasing, constant, or decreasing in time depends on whether $r - \delta$ is greater than, equal to, or less than zero. Furthermore, if $r - \delta > 0$, the *rate of increase* in spending is also increasing with time, and if $r - \delta < 0$, $\lim_{x \to \infty} x(t) = 0$. It follows that we have no edge cases in which one's assets should be expected to grow or shrink asymptotically to a positive size. The $r = \delta$ steady state is unstable. If a fund should grow, it should not stop growing, even once it has grown very large.

Third: Unless

$$\delta > r(1 - \eta), \tag{4}$$

we appear to be led to the conclusion that it is always preferable to invest than to spend. This is, in other words, the condition necessary to avoid the Koopmans (1967) "paradox of the indefinitely postponed splurge". For the purposes of this paper, rather than broach the subject of infinite ethics, we will assume that this condition holds. Note that it does whenever $\eta > 1$, $r > 0$, and $\delta \geq 0$. That is, under the common assumptions that $r > 0$ and $\eta > 1$, agents can be fully patient without entering paradoxical territory.

2.3 The value of patience

Consider an agent whose income grows at the prevailing economic growth rate g, and whose spending on some project grows at g as well. Note that philanthropists giving fixed fractions of their income each period, such as most religious tithers or Giving What We Can members, do this. Such an agent is following the spending schedule that would be optimal if her rate of time preference δ and elasticity of intertemporal substitution η, with respect to spending on the project, are equal to the rate of time preference δ_R and elasticity of intertemporal substitution η_R of a representative agent in the economy at large.

To see this, recall from the Ramsey (1928) formula that $r = \delta_R + \eta_R g$. Rearranging, we have

$$g = \frac{r - \delta_R}{\eta_R}. \tag{5}$$

If $\delta = \delta_R$ and $\eta = \eta_R$, therefore, the agent's spending grows at rate $(r - \delta)/\eta$, as recommended in Proposition 1.

To illustrate the importance of pure time preference in this context, let us now assume that $\eta = \eta_R$ and determine the patient payoff to spending according to some time preference rate $\tilde{\delta} \geq \delta$.

The patient payoff to spending according to $\tilde{\delta}$ can be found by using $\tilde{\delta}$ in the expression for Proposition 1 to get the $\tilde{\delta}$-optimal spending schedule. Then, substitute this schedule as $x(t)$ into (2) to get

$$\int_0^\infty e^{-\delta t} u\Big(B\, \frac{r\eta - r + \tilde{\delta}}{\eta}\, e^{\frac{r-\tilde{\delta}}{\eta} t}\Big) dt. \tag{6}$$

Observe that this will only be defined if

$$\eta \leq 1 \;\text{ or }\; \tilde{\delta} < r + \delta \frac{\eta}{\eta - 1}. \tag{7}$$

If $\eta > 1$ and $\tilde{\delta}$ is too high, the $\tilde{\delta}$-optimal plan may push the spending rate to 0 quickly enough that, though this produces finite $\tilde{\delta}$-discounted disutility, it produces infinite δ-discounted disutility. Note that $\tilde{\delta} < r + \delta$ is sufficient to avoid this condition.

Proposition 2. ***Payoff to spending according a given time preference rate***

Suppose an agent satisfies the conditions of Proposition 1. Then her payoff to following the $\tilde{\delta}$-optimal spending schedule, for some $\tilde{\delta} \geq \delta$, is

$$U_\delta(B, \tilde{\delta}) = \begin{cases} \frac{B^{1-\eta}(r\eta - r + \tilde{\delta})^{1-\eta}}{\eta^\eta(1-\eta)(\delta\eta - (r-\tilde{\delta})(1-\eta))} - \frac{1}{\delta(1-\eta)}, & \eta \neq 1; \\ \\ \frac{\delta \ln(B\tilde{\delta}) + r - \tilde{\delta}}{\delta^2}, & \eta = 1. \end{cases}$$

Proof. Integrate (6) subject to (7). □

We can now calculate how much of her budget a patient agent should be willing to give up to move from the $\tilde{\delta}$-optimal to the δ-optimal spending schedule.

Proposition 3. ***WTP for patience***
Suppose an agent satisfies the conditions of Proposition 1. Then, in order to spend her resources as would be optimal given time preference rate δ as opposed to $\tilde{\delta} \geq \delta$, she is willing to give up the following fraction of her budget:

$$1 - (r\eta - r + \tilde{\delta})(r\eta - r + \delta)^{\frac{\eta}{1-\eta}}(\delta\eta - (r - \tilde{\delta})(1-\eta))^{\frac{-1}{1-\eta}}, \ \eta \neq 1;$$
$$1 - \exp\left(1 + \ln\left(\frac{\tilde{\delta}}{\delta}\right) - \frac{\tilde{\delta}}{\delta}\right), \ \eta = 1.$$

Proof. From Proposition 2, find $\tilde{B} : U_\delta(\tilde{B}, \tilde{\delta}) = U_\delta(B, \delta)$. The formulas above take the form $1 - B/\tilde{B}$. □

This fraction is decreasing in η, as shown numerically in the online appendix accompanying this paper.

Concretely, suppose $r = 5\%$. Then the value achieved by spending according to time preference rate $\tilde{\delta} = 2\%$, by the lights of time preference rate δ, is equal to the value achieved by giving up the following budget-fractions but spending the remaining budget according to time preference rate δ:

η	WTP given $\delta = 0.1\%$	WTP given $\delta = 0.5\%$
0.99	$1 - 4.0 \times 10^{-13}$	0.84
1	$1 - 1.1 \times 10^{-7}$	0.80
1.01	$1 - 1.8 \times 10^{-5}$	0.77
1.25	0.58	0.29
2	0.14	0.07

As we can see, a patient agent errs substantially by adopting spending growth rate g. That is, implicitly spending according to spending rate $\delta_R \approx 2\%$ is a mistake she should be willing to give up a substantial part of her budget to avoid. Furthermore, this willingness to pay is highly sensitive to the values of η and δ. It is most extreme for low values of η and δ: in this case it is almost her entire budget. Even when $\eta = 2$ and $\delta = 0.5\%$, however, following the "rule of thumb" strategy of spending proportionally to income is tantamount to a loss of about 7% of her resources.

As noted above, philanthropists with expressed ethical commitments to intergenerational equity sometimes seem to spend as if they endorsed pure time preference. A cynical economist's interpretation of this phenomenon would be that such philanthropists know what they are doing, and that their behavior reveals that do have pure time preference; indeed, Alexander (2013) reports Robin Hanson taking this position. To the extent that their impatient spending behavior is simply mistaken, however, the result above is a first approximation of true patient philanthropists' willingness to pay (even if they do not yet know it) for better economic reasoning about the discounting problem confronting them.[1]

Furthermore, large philanthropists face a choice between holding their capital in a foundation and holding it privately or in a trust. In the United States, contributions to foundations are tax-exempt, as are the capital gains their assets earn. Foundations must disburse at least 5% of their assets per year, however, effectively requiring them to act impatiently. Trusts are not tax-exempt but are not subject to such a requirement. An analysis like the above can inform large patient philanthropists about how significant the tax advantage to a foundation must be to justify this loss of spending flexibility.

Donor-advised funds, or DAFs, are foundations to which small donors can contribute tax-deductibly. The DAF invests each donor's funds as the donor requests, and it disburses the invested funds to nonprofits of the donor's choosing and on the donor's schedule. The DAF as a whole must disburse at least 5% of its funds per year, but most donors disburse far more quickly (Andreoni (2018)), and a small donor's individual account constitutes only a small fraction of the total (at least for a long time). Small donors can

[1] Having now personally discussed the issue with many philanthropists, including some with assets in the nine figures, I can anecdotally report that they have not thought about discounting in this way, and do value work along these lines. Indeed, some are indirectly funding this research.

therefore use DAFs to act patiently without incurring a tax penalty. As Andreoni argues, the tax deductibility of contributions to DAFs therefore act in part as subsidies for patience, and may be inefficient from an impatient policymaker's perspective.

Some, e.g. Reich (2018), go beyond arguing against tax-deductibility, however, and advocate for requiring trusts or individual DAF accounts to disburse more quickly, with an annual disbursement minimum or by requiring full disbursement within the donor's lifetime. Likewise, in the United Kingdom, where not even foundations face a disbursement minimum, there is extensive discussion about whether to impose one; see Pharoah and Harrow (2010) for an overview. The primary argument in favor is straightforward: in a democracy, voters should have some say in how philanthropic funds are spent, including perhaps in the schedule on which they are spent. In light of the above estimates, imposing a disbursement minimum may be a highly inefficient compromise between allowing philanthropists full control over their giving and fully expropriating their funds to the state. That is, patient philanthropists may be willing to pay a very large one-time tax in order to avoid the disbursement requirement, which an impatient policymaker would also prefer.

3 Free-riding and crowding out

3.1 Motivation and framework

The model above allows us to determine the optimal spending policy regarding the provision of a good for which there is only one purchaser. It applies, for instance, to the schedule on which an individual should allocate her private spending, or on which a philanthropist only interested in funding an esoteric project should allocate his spending on that project. When one is a philanthropist providing a public good to which others contribute, however, one must consider the ways in which one's own funding affects the behavior of the good's other funders. In particular, when one is a patient philanthropist, one must remember that investment for future spending can induce less patient funders to spend more quickly.

As we will see, intertemporal free-riding and crowd-out concerns can motivate substantially different—and, in particular, even "more patient"—behavior from a patient philanthropist than is optimal in the single-funder context. In particular, a patient philanthropist often does best in the presence of impatient funders to invest *all* his resources, for some period, and then to spend on an exponential schedule resembling the single-funder schedule determined above.

Throughout the results below, we will posit a single impatient party and a single patient party. We will denote the impatient party's budget at time t by $B_{I,t}$, with $B_I \triangleq B_{I,0}$; the patient party's budget at time t by $B_{P,t}$ with $B_P \triangleq B_{P,0}$; the impatient time preference rate by δ_I; and the patient time preference rate by $\delta_P < \delta_I$. We will assume that these rates satisfy conditions (4) and (7), with δ_P as δ and δ_I as $\tilde{\delta}$.

We might interpret $\delta_I - \delta_P$ as the portion of the impatient time preference rate consisting of pure time preference. Alternatively, we might hold that the patient party is a philanthropist aiming to supplement the consumption of a lineage of generations, without favoring earlier generations to later ones, whereas the lineage-members themselves exhibit imperfect intergenerational altruism and so discount in part for reasons of mortality risk. Or we might say that the patient philanthropist internalizes the present generation's concern for future generations, but additionally places some weight on the future generations themselves.[2]

At every moment $t \geq 0$, the players observe the spending history $\{(x_I(s), x_P(s))\}_{s<t}$ and independently choose spending rates $x_I(t)$ and $x_P(t)$ respectively.[3] The patient utility function is then given by

$$U_P = \int_0^\infty e^{-\delta_P t} u(x_I(t) + x_P(t)) dt, \tag{8}$$

where $u(\cdot)$ is an isoelastic function parametrized by η, as before.

3.2 Response to a naive impatient funder

Given a patient spending schedule (or history-dependent spending policy) x_P, let us define two ways in which the impatient might respond.

Definition 1. *The impatient funder is* ***strategic*** *if she chooses spending schedule $x_I(t)$ so as to maximize*

$$U_I = \int_0^\infty e^{-\delta_I t} u(x_I(t) + x_P(t)) dt \ : \ \int_0^\infty e^{-rt} x_I(t) dt \leq B_I.$$

[2] Bernheim (1989) shows that these scenarios are formally equivalent.

[3] For a model in which actors with different discount rates jointly set spending rates over time, see Millner and Heal (2018).

Definition 2. *The impatient funder is **naive** if she chooses spending schedule $x_I(t)$ so as to maximize*

$$\tilde{U_I} = \int_0^\infty e^{-\delta_I t} u(x_I(t))dt \ : \ \int_0^\infty e^{-rt} x_I(t)dt \leq B_I,$$

with $x_I(t)$, rather than $x_I(t) + x_P(t)$, as the argument of her flow utility function.

As one might intuit, and as we will later see, the presence of the patient funder may motivate a strategic impatient funder to spend her budget more quickly, in anticipation of the patient funder's future spending. An assumption of this sort is plausible if the impatient party is the patient philanthropist's beneficiary—that is, if the public good to which they are both contributing is the impatient party's consumption—and especially so if the beneficiary can borrow against the philanthropist's future transfers.

If δ_P is sufficiently low, however, the patient party may plan to invest most or all his resources for centuries before spending them for the benefit of the current generation's distant descendants. Borrowing against this far-future income is typically impossible, and we might consider it implausible that the current generation would alter its spending and bequest decisions on the basis of such a distant prospect. Furthermore, when the patient philanthropist's spending partner is simply a less patient philanthropist, we must remember that philanthropy is often notoriously unsophisticated, or motivated by "warm glow"—the size of the contributor's own contributions—rather than by altruistic impact (see e.g. Andreoni (1990)).

Let us therefore begin by determining patient-optimal spending behavior in the presence of a "naive" impatient funder.

Proposition 4. ***Patient spending given a naive impatient funder*** *In the presence of a naive impatient funder, the patient funder does best to follow spending schedule*

$$x_P(t) = \begin{cases} 0, & t < t^*; \\ \left(B_I e^{\frac{r-\delta_I}{\eta}t^*} + B_P e^{rt^*}\right) \frac{r\eta - r + \delta_P}{\eta} e^{\frac{r-\delta_P}{\eta}(t-t^*)} - B_I \frac{r\eta - r + \delta_I}{\eta} e^{\frac{r-\delta_I}{\eta}t}, & t \geq t^*, \end{cases},$$

where

$$t^* = \max\left(0, \ln\left(\frac{B_I}{B_P} \frac{\delta_I - \delta_P}{r\eta - r + \delta_P}\right) \frac{\eta}{r\eta - r + \delta_I}\right).$$

Proof. See Appendix A.2. □

As we can see, the patient party does best to invest *all* his resources as long as his share of total resources is sufficiently low: in particular, as long as

$$\frac{B_{P,t}}{B_{I,t}+B_{P,t}} \leq \frac{\delta_I - \delta_P}{r\eta - r + \delta_I}. \tag{9}$$

The intuition is straightforward. If the impatient party controls a large enough share of total resources, the impatient-optimal spending rate at which to spend her own budget may be higher not just than the patient-optimal rate at which to spend her budget, but than the patient-optimal rate at which to spend the collective budget. If so, any spending by the patient party would, on his view, increase the extent to which they are collectively over-spending. He should only begin spending once the impatient party's share of the collective budget has shrunk enough that even spending it impatient-optimally constitutes underspending the collective budget, from the patient perspective.

3.3 Interaction with a strategic impatient funder

If the impatient funder is strategic, the actors' spending problems take the form of a dynamic game. There is already a substantial literature on dynamic public goods contribution games, but none of it yet appears to have considered the implications of differences in time preference. The framework used here is of course designed to introduce such differences. In fact it isolates the effects of differences in time preference by positing that they are the funders' *only* preference differences; funders do not have the opportunity to spend on private goods which only they value, as they are typically assumed to have. We therefore hold fixed the size of the budget each party contributes to the public good, thereby eliminating the primary concern of the literature on public good contribution.

[Something about the strategy space, reference Stinchcombe and that existing dynamic public goods work in continuous time]

Definition 3. *A* ***defection equilibrium*** *of a dynamic public good contribution game among funders with time preferences $\delta_I > \delta_P$ is a subgame-perfect equilibrium in which $x_I(t) > 0 \implies x_P(s) = 0\ \forall s \leq t$.*

That is, a defection equilibrium is one in which the patient party defects from funding the public good until after the impatient party has permanently stopped spending. In equilibrium, this will entail waiting to spend until after the impatient party has disbursed all her resources; that is, until after $t^* = \inf(\{t : B_{I,t} = 0\})$. [Comment: this is an equilibrium satisfying that property from Bergstrom et al.]

Proposition 5. ***Existence and uniqueness of defection equilibrium***

In a two-player dynamic public good contribution game where flow utility is isoelastic in collective spending and the players have different time preference rates $\delta_I > \delta_P$, there is a unique defection equilibrium, in which the players adopt spending rates

$$\tilde{x}_I(t) = \begin{cases} B_I \, \frac{Z}{Z-1} \, \frac{r\eta - r + \delta_I}{\eta} \, e^{\frac{r-\delta_I}{\eta} t}, & t \leq t^*; \\ 0, & t > t^* \end{cases}$$

and

$$\tilde{x}_P(t) = \begin{cases} 0, & t \leq t^*; \\ B_P \, Z^{\frac{r - r\eta - \delta_P}{r - r\eta - \delta_I}} \, \frac{r\eta - r + \delta_P}{\eta} \, e^{\frac{r-\delta_P}{\eta} t}, & t > t^* \end{cases}$$

respectively, where

$$t^* = \ln(Z) \frac{\eta}{r\eta - r + \delta_I}$$

and

$$Z = \begin{cases} 1 - \frac{B_I}{B_P} \, \frac{r\eta - r + \delta_I}{r\eta - r + \delta_P} \left(\frac{r\eta - r + \delta_P + (\delta_I - \delta_P)\eta}{r\eta - r + \delta_I} \right)^{\frac{1}{1-\eta}}, & \eta \neq 1; \\ \\ \left(1 - \exp\left(\ln\left(\frac{B_I \delta_I}{B_P \delta_P} \right) - \frac{r - \delta_P}{\delta_I} - 1 \right) \right)^{-1}, & \eta = 1. \end{cases}$$

Proof. See Appendix A.3. □

Let us call

$$\tilde{x}(t) \triangleq \tilde{x}_I(t) + \tilde{x}_P(t) \tag{10}$$

the "defection spending schedule". It follows an impatient-optimal spending schedule for $t \leq t^*$ and a patient-optimal spending schedule for $t > t^*$, and is therefore inefficient. The impatient party is indifferent regarding marginal reallocations of resources from $t_1 < t^*$ to $t_2 \in (t_1, t^*]$, whereas the patient party strictly prefers them. Likewise the patient party is indifferent regarding marginal reallocations of resources from $s_2 > t^*$ to $s_1 \in (t^*, s_2)$, whereas the impatient party strictly prefers them. If the parties could coordinate, therefore, they could achieve a Pareto improvement by shifting spending toward t^* from both sides.

As we will now see, however, coordination is not necessary to achieve efficiency.

Proposition 6. ***Efficient equilibria given a strategic beneficiary***
Every efficient Pareto improvment to the defection spending schedule can be obtained in a subgame-perfect equilibrium.

Proof. See Appendix A.4. □

By contrast, consider the folk wisdom that, in a dynamic public good contribution game, the best equilibrium payoffs approach efficiency only when the players' discount rates are sufficiently low. Proposition 6 demonstrates that this intuition does not apply when all preference heterogeneity is due to differences in time preference. Even when players' discount rates are high, or the difference between them is large, there are efficient equilibria.

3.4 The value of patience

Proposition 7. ***WTP for patience given an impatient funder***
TO FILL IN

Proof. See Appendix A.5. □

Proposition 8. ***Payoff ratio to patience given an impatient funder***

TO FILL IN

Proof. See Appendix A.6. □

4 Contribution matching

4.1 Motivation and framework

The continuum of efficient equilibria identified in Proposition 6 is a dynamic conditional contribution scheme, in which two (presumably at least relatively large) parties strategically condition their contributions on each other's contribution histories.

In the face of smaller donors, it is common for large philanthropists not simply to contribute to public goods directly, but to encourage others to do so by explicitly matching their contributions in some proportion. When the large philanthropist is intent on contributing a fixed budget to his chosen cause, smaller impact-minded donors must worry that the matching is illusory; funds not used for contribution matching will ultimately be spent on the cause nonetheless (Kaufman (2015)). Here, however, we will posit that the large philanthropist has a lower rate of time preference δ_P than the small donors' δ_I. Impatient donors therefore do have a counterfactual impact on the patient philanthropist's behavior: funds not contributed at some time t will still be spent on the same good, but not on the same schedule.

Throughout this section, we will assume that the impatient party (the "donor") maximizes her utility taking the matching policy of the patient party (the "philanthropist") as given. Equivalently, and perhaps more precisely, we will assume that the impatient party is not a single agent but a continuum of uncoordinated agents with identical preferences. That is, given an equilibrium in which a representative donor spends according to schedule $x_I(t)$ and her funds are matched accordingly, she can consider the value only of marginal shifts in resources across periods. She cannot consider shifts large enough to affect the affordability of the philanthropist's matching policy.

For simplicity (and realism), we will restrict our attention to linear matching policies. Note that a linear matching policy at some ratio $M : 1$ is equivalent to a policy subsidizing the provision of the good by setting its price at $f = 1/(1 + M)$, where its price without the subsidy is normalized to 1.

4.2 Matching under discretion

In the philanthropic context, we must always have $f(t) \leq 1$. Before solving for the optimal price policy in this setting, however, let us consider the simpler case without this constraint. That is, let us consider the more familar problem of a patient policymaker, who can both tax and subsidize others' spending.

Proposition 9. ***Optimal price schedule when taxation is feasible***
If the patient party can both tax and subsidize spending, he maximizes δ_P*-*

discounted utility by setting the effective price schedule

$$\hat{f}(t) = e^{(\delta_P - \delta_I)t} \frac{B_I}{B_I + B_P} \Big(1 + \frac{\delta_I - \delta_P}{r\eta - r + \delta_P} \eta\Big).$$

Proof. See Appendix A.7. □

Intuitively, a price schedule of the form $Ae^{-(\delta_I - \delta_P)t}$, for some A, lowers the beneficiary's effective discount rate by $\delta_I - \delta_P$. Setting A as high as is feasible thus produces the δ_P-optimal spending plan for the collective budget. For each t, therefore, $\hat{f}(t)$ also produces the δ_P-optimal spending plan for the budget remaining at t; that is, the price schedule is dynamically consistent. This result stands as a happy exception to the fact, well-known in the optimal taxation literature, that policymakers often cannot implement their first-best tax and subsidy policies over time without the power to commit (see e.g. Klein and Rios-Rull (2003); Benhabib and Rustichini (1997)).

Proposition 9 may offer useful advice for a patient policymaker in the setting described. Of course, however, a philanthropist cannot set $f(t) > 1$; a philanthropist cannot tax. If B_P/B_I is large enough that $\hat{f}(0) \leq 1$, this constraint is never binding. (Observe that $\hat{f}(t)$ is decreasing in t.) Otherwise, however, the constraint does of course bind, so the problem changes as follows.

Given a price schedule f, let us define

$$x_f \triangleq \arg\max_{x_I} \int_0^\infty e^{-\delta_I t} u\Big(\frac{x_I(t)}{f(t)}\Big) dt : \int_0^\infty e^{-rt} x_I(t) dt \leq B_I. \tag{11}$$

That is, x_f is the price schedule that maximizes the donor's utility given price schedule f.

If the philanthropist can commit to a subsidy plan at time 0, his problem is then to announce, at 0, a schedule $f : f(t) \leq 1$ of prices for the good such that

$$\int_0^\infty e^{-rt}(1 - f(t)) x_f(t) dt \leq B_P, \tag{12}$$

and there is no $\tilde{f}(t)$ satisfying this budget constraint such that $U(\tilde{f}) > U(f)$, where

$$U(f) \triangleq \int_0^\infty e^{-\delta_P t} u\Big(\frac{x_f(t)}{f(t)}\Big) dt. \tag{13}$$

If the philanthropist cannot commit, however, we must explicitly posit not just a price schedule but a history-dependent price policy F, and this must be dynamically consistent. Informally, that is, we must require for every possible price history up to t that, given that the philanthropist will obey F after t, it is not profitable for the philanthropist to deviate from F at t. The philanthropist's problem is then to announce, at 0, a price policy F such that, given $x_{I,F}$, the budget and dynamic consistency constraints are both satisfied, and there is no $\tilde{F}$ satisfying both constraints such that $U(\tilde{F}) > U(F)$.

Proposition 10. ***Discretionary price policy***
TO FILL IN

Proof. See Appendix A.8. □

4.3 Matching under commitment

Proposition 11. ***Optimality of discretionary pricing when*** $\eta = 1$
TO FILL IN

Proof. See Appendix A.9. □

Proposition 12. ***Sub-optimality of discretionary pricing when*** $\eta \neq 1$

TO FILL IN

Proof. See Appendix A.10. □

4.4 The value of patience

Proposition 13. ***WTP for patient contribution matching***
TO FILL IN

Proof. See Appendix A.11. □

Proposition 14. ***Payoff ratio to patient contribution matching***
TO FILL IN

Proof. See Appendix A.12. □

5 Expropriation

6 Fleeting opportunities

7 Applications

8 Conclusion

The incomplete model developed here suggests that many philanthropists could fulfill their own values substantially more effectively by spending more slowly. It is not clear whether further modeling would weaken or strengthen this suggestion, but we can at least conclude from our model so far that some arguments given in favor of giving now are mistaken. In short, patient philanthropists should reconsider their decision to spend as quickly as they do—and philanthropically-concerned economists should consider that the problem of patience and philanthropy is important, decision-relevant, and amenable to further fruitful exploration.

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Appendix

A.1 Proof of Proposition 1

Let

$$y(t) \triangleq e^{-rt}x(t) \tag{14}$$

denote the resources allocated at time 0 for investment until, followed by spending at, t. Let

$$v_t(y(t)) \triangleq e^{-\delta t}u(e^{rt}y(t)) \tag{15}$$

denote the discounted flow utility at t from allocation $y(t)$. Finally, let us assume, with no substantive loss of generality, that y contains no removable discontinuities.

Since utility in spending is time-additive, differentiable, and strictly concave, allocation y maximizes utility iff, for some constant k,

$$\begin{aligned} v'(y(t)) &= \frac{\partial}{\partial[y(t)]}\Big[e^{-\delta t}\frac{(e^{rt}y(t))^{1-\eta}-1}{1-\eta}\Big] = \lambda \ \forall t,\ \eta \neq 1; \\ &= \frac{\partial}{\partial[y(t)]}\Big[e^{-\delta t}\ln(e^{rt}y(t))\Big] = \lambda \ \forall t,\ \eta = 1. \end{aligned}$$

Taking the derivative and rearranging, we have

$$y(t) = \lambda^{\frac{-1}{\eta}} e^{\frac{r-r\eta-\delta}{\eta}t}. \tag{16}$$

Subjecting this resource allocation to the budget constraint, we have

$$\int_0^\infty \lambda^{\frac{-1}{\eta}} e^{\frac{r-r\eta-\delta}{\eta}t} dt = B; \tag{17}$$

$$\lambda = \Big(B\Big(\frac{r\eta - r + \delta}{\eta}\Big)\Big)^{-\eta}. \tag{18}$$

Substituting (18) into (16), and observing that $x(t) = e^{rt}y(t)$, we have

$$x(t) = B\Big(\frac{r\eta - r + \delta}{\eta}\Big)e^{\frac{r-\delta}{\eta}t}. \tag{19}$$

A.2 Proof of Proposition 4

As in the proof of Proposition 1 (Appendix A.1), let

$$y_P(t) \triangleq e^{-rt} x_P(t) \tag{20}$$

denote the resources the patient party allocates at time 0 for investment until, followed by spending at, t. Let $y_I(t)$ be defined likewise, and let $y(t) \triangleq y_P(t) + y_I(t)$. Given that the impatient party follows allocation y_I, let

$$v_{P,t}(y(t)) \triangleq e^{-\delta_P t} u(e^{rt} y(t)) \tag{21}$$

denote the patient party's discounted flow utility at t from allocation $y_P(t)$. Finally, let us again assume, with no substantive loss of generality, that y_P and y_I contain no removable discontinuities.

From Proposition 1, the impatient party's spending schedule is

$$y_I(t) = B_I \frac{r\eta - r + \delta_I}{\eta} e^{\frac{r - r\eta - \delta_I}{\eta} t}, \tag{22}$$

independently of y_P. Furthermore, the patient party's discounted flow utility in the collective allocation $y(t)$ is time-additive, differentiable, strictly increasing, and strictly concave at each time t. Taking $y_I(t)$ as given, therefore, the patient party maximizes his utility by setting $y_P(t)$ such that he is indifferent to marginal resource reallocation across times to which he is allocating resources at a positive rate, and weakly prefers marginal resource allocation to these times to marginal resource allocation to other times. That is, differentiating (21),

$$\begin{aligned} \lambda_t(y_P(t), y_I(t)) &\triangleq \frac{\partial}{\partial[y_P(t)]}\Big[v_{P,t}(y_P(t) + y_I(t))\Big] \\ &= e^{(r - r\eta - \delta_P)t}(y_P(t) + y_I(t))^{-\eta} \\ &= \lambda^* > 0 \qquad\qquad \text{if } y_P(t) > 0; \\ &\le \lambda^* \qquad\qquad\quad\ \ \text{if } y_P(t) = 0. \end{aligned} \tag{23}$$

Substituting (22) into (23), we have that if $y_P(t) = 0$,

$$\lambda_t = \left(B_I \frac{r\eta - r + \delta_I}{\eta}\right)^{-\eta} e^{(\delta_I - \delta_P)t}. \tag{24}$$

As we can see, if $y_P(t) = 0$, λ_t is strictly increasing in t. It follows from (23) that, if $y_P(t) > 0$ for some t, $y_P(s) > 0\ \forall s > t$. That is, there is some t^* such that $y_P(t) = 0\ \forall t < t^*$ and $y_P(t) = 0\ \forall t > t^*$.

Thus $v'_{P,t}(y(t)) = \lambda^*$ is constant for all $t > t^*$. This implies that following t^*, the collective allocation $y(t)(t > t^*)$ constitutes the patient-optimal allocation of the collective budget allocated to $t > t^*$.

This leaves us with two cases.

If $t^* = 0$, then $\lambda_t = \lambda^*\ \forall t$, so y constitutes the patient-optimal allocation of the collective budget. The impatient allocation rate of B_I at $t = 0$ must therefore not be greater than the patient allocation rate of the collective budget at $t = 0$. That is,

$$y_I(0) = B_I \frac{r\eta - r + \delta_I}{\eta} \leq (B_P + B_I)\frac{r\eta - r + \delta_P}{\eta}. \tag{25}$$

If $t^* > 0$, note first that $y_P(t)$ must be continuous. If there were some $\tilde{t}$ at which $y_P(t)$ were discontinuous, then, since $y_I(t)$ is continuous, $y(t) = y_P(t) + y_I(t)$ would also be discontinuous at $\tilde{t}$. Because $v_{P,t}(y(t))$ is continuous in t and $y(t)$, it too would then be discontinuous at $\tilde{t}$. The patient party would then be able to increase his utility by reallocating marginal funds from $\tilde{t}$ to $\tilde{t} - \epsilon$ or $\tilde{t} + \epsilon$, for some sufficiently small $\epsilon > 0$.

In particular, y_P is continuous at t^*. Since $y_P(t) = 0\ \forall t < t^*$, it follows that $y_P(t^*) = 0$.

Furthermore, since $\lambda_t(y_P(t), y_I(t))$ is continuous in $y_P(t)$, $y_I(t)$, and t, and since $\lambda_t(y_P(t), y_I(t)) = \lambda^*\ \forall t > t^*$, we now have $\lambda_{t^*} = \lambda^*$. Thus $y(t^*) = y_I(t^*)$ constitutes the patient-optimal allocation rate of the collective resources remaining at t^*.

That is,

$$B_I \frac{r\eta - r + \delta_I}{\eta} e^{\frac{r - r\eta - \delta_I}{\eta} t^*} = \Big(B_P + B_I e^{\frac{r - r\eta - \delta_I}{\eta} t^*}\Big)\frac{r\eta - r + \delta_P}{\eta}. \tag{26}$$

Rearranging, we have

$$t^* = \ln\Big(\frac{B_I}{B_P}\frac{\delta_I - \delta_P}{r\eta - r + \delta_P}\Big)\frac{\eta}{r\eta - r + \delta_I}. \tag{27}$$

Now, multiplying both sides of (26) by $e^{\frac{r\eta - r + \delta_I}{\eta} t^*}$ and substituting (27) for t^*, we have

$$B_I \frac{r\eta - r + \delta_I}{\eta} = \Big(B_P\Big(\frac{B_I}{B_P}\frac{\delta_I - \delta_P}{r\eta - r + \delta_P}\Big) + B_I\Big)\frac{r\eta - r + \delta_P}{\eta}$$
$$> (B_P + B_I)\frac{r\eta - r + \delta_P}{\eta}, \tag{28}$$

because, from (27),

$$t^* > 0 \implies \frac{B_I}{B_P}\frac{\delta_I - \delta_P}{r\eta - r + \delta_P} > 1. \tag{29}$$

The inequality on $y_I(0)$ provided by (25) and (28) thus characterizes whether $t^* = 0$ or $t^* > 0$. In particular, solving it for t^*, we have

$$t^* = \begin{cases} 0 & \frac{B_I}{B_P}\frac{\delta_I - \delta_P}{r\eta - r + \delta_P} \leq 1 \\ \ln\left(\frac{B_I}{B_P}\frac{\delta_I - \delta_P}{r\eta - r + \delta_P}\right)\frac{\eta}{r\eta - r + \delta_I} & \frac{B_I}{B_P}\frac{\delta_I - \delta_P}{r\eta - r + \delta_P} > 1 \end{cases} \tag{30}$$

which reduces to

$$t^* = \max\Big(0, \ln\Big(\frac{B_I}{B_P}\frac{\delta_I - \delta_P}{r\eta - r + \delta_P}\Big)\frac{\eta}{r\eta - r + \delta_I}\Big). \tag{31}$$

Finally, observe that the collective budget at t^* is

$$B_I e^{\frac{r - \delta_I}{\eta} t^*} + B_P e^{r t^*} \tag{32}$$

in either case, and recall that $x_P(t)$ will fill the gap between $x_I(t)$ and the patient-optimal spending rate of the collective budget following t^*. It follows immediately that

$$x_P(t) = \begin{cases} 0 & t < t^* \\ \Big(B_I e^{\frac{r-\delta_I}{\eta} t^*} + B_P e^{r t^*}\Big)\frac{r\eta - r + \delta_P}{\eta} e^{\frac{r - \delta_P}{\eta}(t - t^*)} - B\frac{r\eta - r + \delta_I}{\eta} e^{\frac{r - \delta_I}{\eta} t} & t \geq t^* \end{cases}.$$

A.3 Proof of Proposition 5

A.4 Proof of Proposition 6

A.5 Proof of Proposition 7

A.6 Proof of Proposition 8

A.7 Proof of Proposition 9

Facing price schedule $f(t)$, let us denote the donor's budget allocation $y_f(t)$. As usual, let us assume that $y_f(t)$ contains no removable discontinuities. The allocation is then impatient-optimal iff for some constant λ,

$$\frac{\partial}{\partial[y_f(t)]}\left[e^{-\delta t}\frac{\left(\frac{e^{rt}y_f(t)}{f(t)}\right)^{1-\eta}}{1-\eta}\right] = \lambda \ \forall t. \tag{33}$$

Taking the derivative, substituting $\hat{f}$ for f, and rearranging, we have

$$y_{\hat{f}}(t) = \lambda^{\frac{-1}{\eta}} e^{\frac{r-r\eta+\delta_P\eta-\delta_I\eta-\delta_P}{\eta}t}\left(\frac{B_I}{B_I+B_P}\right)^{\frac{\eta-1}{\eta}}\left(1+\frac{\delta_I-\delta_P}{r\eta-r+\delta_P}\eta\right)^{\frac{\eta-1}{\eta}}. \tag{34}$$

Subjecting this resource allocation to the donor's budget constraint, we have

$$\lambda^{\frac{-1}{\eta}}\left(\frac{B_I}{B_I+B_P}\right)^{\frac{\eta-1}{\eta}}\left(1+\frac{\delta_I-\delta_P}{r\eta-r+\delta_P}\eta\right)^{\frac{\eta-1}{\eta}}\int_0^\infty e^{\frac{r-r\eta+\delta_P\eta-\delta_I\eta-\delta_P}{\eta}t}dt = B_I; \tag{35}$$

$$\lambda^{\frac{-1}{\eta}}\left(\frac{B_I}{B_I+B_P}\right)^{\frac{\eta-1}{\eta}}\left(1+\frac{\delta_I-\delta_P}{r\eta-r+\delta_P}\eta\right)^{\frac{\eta-1}{\eta}} = B_I\frac{r\eta-r-\delta_P\eta+\delta_I\eta+\delta_P}{\eta}. \tag{36}$$

Substituting (36) into (34) and remembering that $x(t) = e^{rt}y(t)$, we see that the donor spends according to schedule

$$x_{\hat{f}}(t) = B_I\,\frac{r\eta-r-\delta_P\eta+\delta_I\eta+\delta_P}{\eta}\,e^{\frac{r+\delta_P\eta-\delta_I\eta-\delta_P}{\eta}t}. \tag{37}$$

At each t, collective spending thus equals

$$\frac{x_{\hat{f}}(t)}{\hat{f}(t)} = (B_I+B_P)\frac{r\eta-r+\delta_P}{\eta}e^{\frac{r-\delta_P}{\eta}t}. \tag{38}$$

From Proposition 1, this is the spending schedule which maximizes δ_P-discounted utility given budget constraint B_I+B_P. It follows both that the given price schedule $\hat{f}$ is feasible and there is no patient-preferable feasible price schedule.

A.8 Proof of Proposition 10

Following the notation of Stinchcombe (2013), let $h_{|t} \triangleq \{f(s)\}_{s<t}$ denote the contribution history from 0 to just before t.

A.9 Proof of Proposition 11

A.10 Proof of Proposition 12

A.11 Proof of Proposition 13

A.12 Proof of Proposition 14