Welfare Implications of Accelerating Long-Run Consumption Growth

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Abstract

The value of accelerating consumption growth, via either a growth effect or a level effect, is highly sensitive to our assumptions about the relationship between consumption and welfare. In particular, under the standard assumption of isoelastic utility with a coefficient of relative risk aversion greater than one, an eternal acceleration to consumption growth, however large, creates only finite welfare.

1 Implications under various functional forms

We will assume

- a homogeneous population of constant size $N > 0$,
- an infinite time horizon,
- zero pure time preference, and
- a constant baseline growth rate of $g > 0$.

Without loss of generality, we will say the consumption level at $t = 0$, before any growth or level effects, equals 1.

1.1 Isoelastic utility with $\eta > 1$

If we assume iseoelastic utility with inverse elasticity of intertemporal substitution $\eta > 1$, we find that the welfare gain from permanently accelerating consumption growth—i.e. from producing a permanent growth effect—is finite:

$$N \int_0^\infty \Big(\frac{e^{kgt(1-\eta)}}{1-\eta} - \frac{e^{gt(1-\eta)}}{1-\eta} \Big) dt < \infty \,, \tag{1}$$

where $g > 0$ denotes the original growth rate and $k > 1$ denotes the (constant) factor by which the growth rate is accelerated.

Calculation:

$$\begin{aligned} & N \int_0^\infty \Big(\frac{e^{kgt(1-\eta)}}{1-\eta} - \frac{e^{gt(1-\eta)}}{1-\eta}\Big) dt \\ =& \frac{N}{1-\eta}\Bigg[\frac{e^{kg(1-\eta)t}}{kg(1-\eta)} - \frac{e^{g(1-\eta)t}}{g(1-\eta)}\Bigg]_0^\infty \\ =& \frac{N}{1-\eta}\Big[\frac{1}{g(1-\eta)} - \frac{1}{kg(1-\eta)}\Big] \\ =& \frac{N(k-1)}{kg(\eta-1)^2}. \end{aligned} \tag{2}$$

As we can see, the welfare gain is finite.

Furthermore, as k increases, the welfare gain from accelerating growth by factor k converges to

$$\frac{N}{g(\eta-1)^2}\,, \tag{3}$$

which is also finite (and, naturally, equals the welfare gain from achieving satiety from $t = 0$ onward).

1.1.1 Exploring the magnitudes of possible welfare gains

Let us now estimate the possible welfare gains from accelerating growth, under these assumptions, accruing from increases to the future consumption of those already living comfortably (as distinct from the welfare gains that might result from accelerating the elimination of poverty).

Suppose η is as low as 5/4. Also, let \$30,000 denote one unit of consumption, and suppose that the consumption level producing "zero welfare" is \$200/year, or 1/150 of a unit. That is, assume that

$$u(c) = \frac{1}{4}\Big(\Big(\frac{1}{150}\Big)^{-\frac{1}{4}} - c^{-\frac{1}{4}}\Big). \tag{4}$$

Note that the added constant of

$$\frac{1}{4}\cdot\Big(\frac{1}{150}\Big)^{-\frac{1}{4}} \approx 0.87 \tag{5}$$

—which equals the welfare level $u(c)$ as $c \to \infty$—does not affect the welfare difference calculations above, since it would be added and then subtracted.

Then the welfare level currently enjoyed at \$30,000/year, as a fraction of the welfare upper bound, is

$$\frac{\frac{1}{4}\Big(\Big(\frac{1}{150}\Big)^{-\frac{1}{4}} - 1\Big)}{\frac{1}{4}\Big(\frac{1}{150}\Big)^{-\frac{1}{4}}} \approx 0.71. \tag{6}$$

That is, someone consuming \$30,000/year is already typically about 71% of the way from nonexistence to the welfare level she would approach if she had all the wealth in the world—or more, since $\eta = 5/4$ would generally be regarded as an optimistic assumption. On reflection, the conclusion that \$30,000/year typically offers welfare over 71% of the way to satiety strikes me as realistic. Is there any amount of wealth which it would be prudentially rational for a typical middle-class individual in the developed world to risk a 29% chance of death in order to attain?

Now suppose also that the baseline growth rate g is low, at 0.016. Then the welfare produced by bringing the entire population to satiety forever, as given by (3), is $1000N$. Since the "satiety level" here offers each individual approximately 0.87 units of welfare, by (5), the benefit of infinite growth—of bringing the entire population to satiety forever—equals approximately the value of $(1000/0.87)N \approx 1143N$ years of life at satiety. That is, a classical utilitarian should be indifferent between (a) satiating the existing population forever and (b) leaving the existing population's growth path unchanged but creating an equal-sized population, a "parallel earth", which enjoys full satiety and lasts slightly over a millenium.

Consider a (still highly optimistic, but at least somewhat more realistic) intervention: one that multiplies the growth rate by $k = 1.01$ forever. By (2), this produces approximately $9.9N$ units of welfare, which is in turn approximately as valuable as $(9.9/0.87)N \approx 11N$ years of life at satiety. That is, a classical utilitarian should be indifferent between (a) multiplying the growth rate by 1.01 forever and (b) leaving the existing population's growth path unchanged but creating an equal-sized population, a "parallel earth", which enjoys full satiety and lasts slightly over a decade.

1.2 Utility given by $\frac{-1}{1+\ln(c)}$

The above observation is not just an artifact of the fact that welfare is bounded above when $\eta > 1$, since we are integrating without discounting over an infinite horizon. The welfare gain achieved by, for instance, moving permanently from any zero-growth plateau to any higher level is of course infinite regardless of η.

More interestingly, it can simultaneously be the case (a) that utility is bounded above and (b) that accelerating growth forever—i.e. producing a permanent growth effect—produces infinite welfare. Consider the form

$$u(c) = \frac{-1}{1+\ln(c)} \quad (c \geq 1), \tag{7}$$

for instance. It is bounded above, but given $k > 1$,

$$N \int_0^\infty \Big(\frac{-1}{1+\ln(e^{kgt})} - \frac{-1}{1+\ln(e^{gt})} \Big) dt \tag{8}$$

$$= Ng(k-1) \int_0^\infty \frac{t}{1+g(k+1)t+kg^2t^2} dt. \tag{9}$$

This integrand is everywhere positive, and for all $t > 1$ it is strictly greater than

$$\frac{t}{(1 + g(k+1) + kg^2)t^2}, \tag{10}$$

which is uniformly bounded across $t \in [0, 1]$. Furthermore,

$$\frac{Ng(k-1)}{1 + g(k+1) + kg^2} \int_0^\infty \frac{1}{t} dt = \infty. \tag{11}$$

So (9) $= \infty$ as well.

Now let us consider the welfare gains from a one-time positive shock to consumption that compounds indefinitely at rate g: that is, the gains from a *level effect* on the growth path. These gains are finite: given $k > 1$,

$$N \int_0^\infty \Big(\frac{-1}{1 + \ln(ke^{gt})} - \frac{-1}{1 + \ln(e^{gt})} \Big) dt \tag{12}$$

$$= N \ln(k) \int_0^\infty \frac{1}{1 + (2 + \ln(k))gt + g^2 t^2} dt. \tag{13}$$

This integrand is weakly less than

$$\frac{1}{1 + g^2 t^2} \tag{14}$$

for all $t \geq 0$. Furthermore,

$$N \ln(k) \int_0^\infty \frac{1}{1 + g^2 t^2} dt = N \ln(k) \frac{\pi}{2g}, \tag{15}$$

which is finite. So (13) is finite as well.

1.3 Logarithmic utility

If the relationship between consumption and welfare is logarithmic, not only the gains from a growth effect, but even the gains from a level effect, are infinite. Given $k > 1$,

$$N \int_0^\infty \Big(\ln(e^{kgt}) - \ln(e^{gt}) \Big) dt \tag{16}$$

$$= Ng(k-1) \int_0^\infty t \; dt,$$

which is of course infinite.

And

$$N \int_0^\infty \Big(\ln(ke^{gt}) - \ln(e^{gt}) \Big) dt \tag{17}$$

$$= N \int_0^\infty ln(k) \; dt,$$

which is of course infinite as well.

2 Discussion

If we believe that the initial assumptions above are not too problematic, the value of promoting consumption growth is highly sensitive to our guesses about the shape of the long-run relationship between consumption and welfare. In particular, if we think there is a substantial chance that the relationship is generally (something like) isoelastic with $\eta > 1$, we may think that other interventions are more likely to produce very large quantities of long-run value. Some more robustly promising alternatives, being less sensitive to our guess about the functional form we're dealing with here, might then be:

- Increasing long-term population size.
- Reducing risks of long-term shocks to population size, such as extinction.
- Reducing risks of long-term zero or negative growth. (This may serve as a useful characterization of "existential but not extinction" risk.)
- Engaging in blue-sky research about how to produce more welfare per individual than is permitted by "increasing consumption" as we now know it.

Note that tradeoffs between *foregone* growth and the alternatives above must still be considered seriously. It may be, for instance, that population increases push long-term growth ever closer to 0, and that the unbounded *loss* available from *decreasing* growth outstrips the unbounded gains available from increasing population.

Also, note that there may be benefits (or costs) to economic growth beyond those captured directly by human consumption. Growth may increase or decrease extinction risk, for example.