Welfare Implications of Accelerating Long-Run Consumption Growth

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Abstract

The value of accelerating consumption growth is highly sensitive to our assumptions about the relationship between consumption welfare. In particular, under the standard assumption of isoelastic utility with a coefficient of relative risk aversion greater than one, an eternal acceleration to consumption growth, however large, creates only finite welfare.

1 Implications under various functional forms

We will assume

- a homogeneous population of constant size N and
- an infinite time horizon, with zero discounting.

Note that, since we are assuming a constant population size, nothing depends on what consumption level is welfare-equivalent to nonexistence. The welfare gain from a permanent increase to the growth rate always equals the term below.

We can therefore assume without loss of generality that consumption at time zero equals 1.

1.1 Isoelastic utility with $\eta > 1$

If we assume iseoelastic utility parametrized by $\eta > 1$, we find that the welfare gain from permanently accelerating consumption growth is finite:

$$N \int_0^\infty \Big(\frac{e^{kgt(1-\eta)}}{1-\eta} - \frac{e^{gt(1-\eta)}}{1-\eta} \Big) dt < \infty \,,$$

where $g > 0$ denotes the original growth rate and $k > 1$ denotes the (constant) factor by which the growth rate is accelerated.

Calculation:

$$N \int_0^\infty \Big(\frac{e^{kgt(1-\eta)}}{1-\eta} - \frac{e^{gt(1-\eta)}}{1-\eta} \Big) dt$$

$$= \frac{N}{1-\eta}\left[\frac{e^{kg(1-\eta)t}}{kg(1-\eta)} - \frac{e^{g(1-\eta)t}}{g(1-\eta)}\right]_0^\infty$$

$$= \frac{N}{1-\eta}\left[\frac{1}{g(1-\eta)} - \frac{1}{kg(1-\eta)}\right]$$

$$= \frac{N(k-1)}{kg(1-\eta)^2}$$

As we can see, the welfare gain is finite.

Furthermore, as k increases, the welfare gain from accelerating growth by factor k converges to

$$\frac{N}{g(1-\eta)^2},$$

which is also finite (and, naturally, equals the welfare gain from achieving satiety from $t = 0$ onward).

1.2 Utility given by $\frac{-1}{ln(1+c)}$

Remember that the above observation is not *just* an artifact of the fact that welfare is bounded above when $\eta > 1$, since we are integrating without discounting over an infinite horizon. The welfare gain achieved by, for instance, moving permanently from any zero-growth plateau to any higher level is of course infinite regardless of η.

More interestingly, note that it can simultaneously be the case that (a) utility is bounded above and (b) accelerating growth forever produces infinite welfare. Consider the form $u(c) = \frac{-1}{ln(1+c)}$, for instance. It is bounded above, but

$$N \int_0^\infty \Big(\frac{-1}{ln(1+e^{kgt})} - \frac{-1}{ln(1+e^{gt})}\Big)dt, k > 1, g > 0$$

is infinite.

The gains from eternally compounding consumption growth from a *temporary* shock to consumption are still finite, though:

$$N \int_0^\infty \Big(\frac{-1}{ln(k+e^{gt})} - \frac{-1}{ln(1+e^{gt})}\Big)dt, k > 1, g > 0$$

is finite.

1.3 Logarithmic utility

If the relationship between consumption and welfare is logarithmic, not only the gains from an eternal acceleration to growth, but even the consumption-compounding gains from a temporary shock to consumption, are infinite.

Assuming $k > 1$ and $g > 0$,

$$N \int_0^\infty \Big(ln(e^{kgt}) - ln(e^{gt}) \Big) dt$$

$$= Ng(k-1) \int_0^\infty t \; dt,$$

which is of course infinite.

And

$$N \int_0^\infty \Big(ln((ke)^{gt}) - ln(e^{gt}) \Big) dt$$

$$= N \int_0^\infty ln(k^{gt}) dt$$

$$= N \left[\frac{ln^2(k^{gt})}{2ln(k^g)} \right]_0^\infty$$

which is infinite as well.

2 Discussion

If we believe that the initial assumptions above are not too problematic, the value of promoting consumption growth is highly sensitive to our guesses about the shape of the long-run relationship between consumption and welfare. In particular, if we think there is a substantial chance that the relationship is generally (something like) isoelastic with $\eta > 1$, we may think that other interventions are more likely to produce infinite (or, sufficiently "astronomical") value. Some more robustly promising alternatives, being less sensitive to our guess about the functional form we're dealing with here, might then be:

- Increasing long-term population size.
- Reducing risks of long-term shocks to population size, such as extinction.
- Reducing risks of long-term zero or negative growth. (This may serve as a useful characterization of "existential but not extinction" risk.
- Engaging in blue-sky research about how to produce more welfare per individual than is permitted by "increasing consumption" as we now know it.

Note that tradeoffs between *foregone* growth and the alternatives above must still be considered seriously. It may be, for instance, that population increases push long-term growth ever closer to 0, and that the unbounded *loss* available from *decreasing* growth outstrips the unbounded gains available from increasing population.

Also, note that there may be benefits (or costs) to economic growth beyond those captured directly by human consumption. Growth may increase or decrease extinction risk, for example.