Weak Betweenness and Misleading Information

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Abstract

I propose a new axiom of normative decision theory, termed "Weak Betweenness", and show that it is substantively weaker and perhaps more normatively compelling than Betweenness, and thus also Independence. I show that Weak Betweenness is equivalent to an intuitive formalization of the condition that one always accept costless "non-misleading" information. Finally, I observe that if a decision-maker obeys the axioms of rank-dependent utility (RDU) theory, and is further committed to Weak Betweenness, she cannot be anywhere-risk-avoidant.

1   Introduction

When we choose acts, we face uncertainty about which outcomes those acts will yield. In 1944, von Neumann and Morgenstern famously provided a short list of axioms and proved that any agent who obeys those axioms can be represented as maximizing expected utlity (where utlity is a real-valued function, unique up to positive affine transformation, of final outcomes). The claim that to be rational an agent should be representable in this way is known as expected utility theory (EU).

For now over seventy-five years, normative decision theorists have debated this claim. The most widely accepted position is that EU is correct. Another common position is that one of the von Neumann-Morgenstern axioms, termed Independence, should be replaced with some weaker axiom. Such replacements are designed to accommodate reasonable-seeming patterns of behavior that are not (representable as) expected-utility-maximizing.

As Chew and Epstein (1989) testify, and as a review of the subsequent literature confirms, there are two primary means of weakening Independence. The most commonly explored weakening is to an axiom known as Comonotonic Independence. This replacement generalizes expected utility theory to rank-dependent utility theory

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(RDU): a decision theory first sketched by Quiggin (1982) (under the name "anticipated utility theory"), refined by Yaari (1987), and popularized and normatively defended by Buchak (2013) (under the name "risk-weighted expected utility theory").[1] While RDU permits various reasonable-seeming deviations from expected utility maximization, it comes with some undesirable features: in particular, and as this paper explores, it permits certain particularly counterintuitive reactions to information.

The other commonly proposed weakening is to an axiom known as Betweenness. See Chew (1983), Dekel (1986) and Gul (1991)) for examples of decision theories that satisfy the Betweenness axiom.

Comonotonic Independence and Betweenness represent distinct and incompatible weakenings of Independence. Chew and Epstein (1989) show that Betweenness and Comonotonic Independence together entail Independence, so if we accept one weakening of Independence, we must reject the other. Some normative theorists, such as Bottomley and Williamson (forthcoming), reject Independence and, in its place, defend Betweenness rather than Comonotonic Independence. But given the ongoing proposed defenses of RDU, it seems that many are inclined to reject Betweenness as a universally valid constraint. If we are to resolve this debate, it seems best not merely to insist on Betweenness, but rather to find some weaker and more universally acceptable principle of rational behavior.

This paper proves that there is an axiom of normative decision theory, strictly weaker than Betweenness, which nevertheless (in conjunction with Comonotonic Independence) rules out precisely those desired deviations from EU-maximization that most strongly motivate the search for alternative decision theories. This proposed axiom, termed Weak Betweenness, carries a straightforward intuitive meaning and obvious normative appeal.

2 Framework

We will use a slight modification of the Savage (1954) framework, in which an agent faces a set X of possible outcomes and is uncertain about the state of the world.

For simplicity, we will assume that the uncertainty about the state is representable by a standard continuous probability space $\mathscr{S} = (S, \mathcal{A}, \mu)$. We will denote the true state by s^*.

Definition 1. *An <u>act</u> $F \in X^S$ is a function from S to X.*

Definition 2. *An <u>event</u> $\mathcal{E} \in \mathcal{A}$ is a measurable set of states.*

Let Φ be the set of acts under consideration, where

$$\Phi \triangleq \{F \in X^S : |F(S)| < \infty, F^{-1}(x) \in \mathcal{A} \ \forall x \in X\}. \tag{1}$$

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[1]See Diecidue and Wakker (2001) for a precursor to Buchak's normative defense.

That is, for simplicity, let us consider the agent's preferences over all and only acts which realize a finite set of outcomes and which assign outcomes to events.

Definition 3. *A <u>simple act</u> f is an act which assigns the same outcome to all states.*

We will denote simple acts by lowercase letters and acts in general by uppercase letters, with $\phi \subset \Phi$ denoting the set of simple acts.

We will denote the probability distribution over X induced by some act F by $d(F)$, and that induced given some event $\mathcal{E}$ by $d(F|\mathcal{E})$. We will denote the p-mixture of an ordered pair of distributions (d_1, d_2) by $(p, d_1; 1 - p, d_2)$. Note that $\exists H \in \Phi :$ $d(H) = (p, d(F); 1 - p, d(G)) \ \forall F, G \in \Phi \ \forall p \in [0, 1]$. That is, the set of distributions over X inducable by acts under consideration is closed under p-mixture.

We will denote the outcome of act F in state s by $x(F, s)$. For simplicity, we will sometimes denote the outcome of a simple act f by $x(f)$.

Definition 4. *An agent's preferences over a set of acts Γ satisfy the <u>Completeness</u> axiom iff, for any two acts $F, G \in \Gamma$, $F \succsim G$ or $G \succsim F$.*

Definition 5. *An agent's preferences over some set of acts Γ satisfy the <u>Transitivity</u> axiom iff, for any three acts $F, G, H \in \Gamma$ where $F \succsim G$ and $G \succsim H$, $F \succsim H$.*

Proposition 1. *An agent's preferences over ϕ satisfy Completeness and Transitivity iff they can be represented by a utility function over outcomes $u : X \to \mathbb{R}$, unique up to strict monotonic transformation, such that $f \succsim g \iff u(x(f)) \geq u(x(g)) \ \forall f, g \in \phi$.*

A brief proof is given by Mas-Colell et al. (1995), p. 9.

Let I_n denote the first n naturals: $\{1, ..., n\}$. Given a utility function $u(\cdot)$ over outcomes, we will impose an ordering on the image $F(S)$ of each act F, such that

$$i \leq j \iff u(F(S)_i) \leq u(F(S)_j) \ \ \forall i, j \in I_{|F(S)|}. \tag{2}$$

Definition 6. *Given a set X of possible outcomes and a utility function $u : X \to \mathbb{R}$, <u>continuity in outcomes</u> obtains iff, for any two outcomes $x_1, x_2 \in X$ and any $p \in [0, 1]$, $\exists x \in X : u(x) = pu(x_1) + (1 - p)u(x_2)$.*

One might consider the assumption of continuity in outcomes analogous to the assumption of a continuous commodity space in consumer theory, in the presence of continuous preferences over commodities. We will assume continuity in outcomes throughout this paper.

Finally, given a utility function, we will define the following:

Definition 7. *The <u>value</u> of an act F, $v(F) \triangleq u(x(F, s^*))$, is the utility assigned to the outcome of the act in the true state.*

Definition 8. *An act F is <u>admissible</u> with respect to some set of acts $\Gamma \ni F$ iff $F \succsim G \ \forall G \in \Gamma$.*

We will denote the set of acts admissible with respect to Γ by $b(\Gamma)$.

Definition 9. *An agent's preferences over a set of acts Γ satisfy the <u>Continuity</u> axiom iff, for any three acts $F, G, H \in \Gamma$ where $F \succsim G \succsim H$, $\exists p \in [0,1] : G \sim G' \; \forall G' : d(G') = (p, F; 1 - p, H)$.*

Note that if an agent's preferences over Γ satisfy Continuity, then $d(F) = d(G) \implies F \sim G \; \forall F, G \in \Gamma$.

We will assume that a rational agent's preferences over Φ satisfy Completeness, Transitivity, and Continuity.

3   Independence and EU

In addition to the three axioms of rational decision-making defined above, many find the following to have some intuitive appeal.

Definition 10. *An agent's preferences over a set of acts Γ satisfy the <u>Independence</u> axiom iff, for any five acts $F, G, H, K, L \in \Gamma$ where $G \succsim F$, $d(K) = \overline{(p, d(F); 1 - p, d(H))}$, and $d(L) = (p, d(G); 1 - p, d(H))$, $L \succsim K$.*

3.1   Expected utility theory

Proposition 2 (The von Neumann-Morgenstern Utility Theorem). *An agent's preferences over Φ satisfy Completeness, Transitivity, Continuity, and Independence iff they can be represented by a utility function over outcomes $u : X \to \mathbb{R}$, unique up to positive affine transformation, such that $F \succsim G \iff \mathbb{E}[u(x(F, s))] \geq \mathbb{E}[u(x(G, s))]$.*[2]

Informally, an agent's preferences over acts satisfy the von Neumann-Morgenstern axioms if and only if the agent *maximizes expected value*.

Definition 11. <u>*Expected utility theory*</u> *is the claim that rational agents must maximize expected value.*

An alternative axiomatization of expected utility theory can be found in Savage (1954). Rather than assuming that the state space comes endowed with a probability measure, he elicits both a (subjective) probability measure over the state space *and* a utility function over the outcome set, relative to which the agent maximizes expected utility. One of Savage's axioms is termed the "Sure Thing Principle". As Friedman and Savage (1952) show, this principle, once formalized in the context of well-defined

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[2]Note that these expectations will always be well-defined because we are assuming that each act induces only finitely many outcomes. Also, note that von Neumann and Morgenstern's original (1944) proof only covered the case in which $|X|$ itself is finite. For an extension to the infinite case discussed here, see Kreps (1984, ch. 5).

distributions over outcomes, is equivalent to Independence. We will therefore refer exclusively to the latter for the remainder of this paper.[3]

3.2 The debate over Independence: background

Core to the debate over EU is the status of Independence (as testified by, e.g., Friedman and Savage (1952), p. 468).[4]

The case against Independence is most clearly motivated by the observation that certain widely appealing act-preferences are incompatible with Independence.[5] The most famous of these are described by Allais (1953).

Consider the following four acts, as described by their induced distributions over outcomes:

- If A is chosen, the agent receives \$0 with probability 0.01, \$1 million with probability 0.89, and \$5 million with probability 0.10.

- If B is chosen, the agent receives \$1 million for certain.

- If C is chosen, the agent receives \$0 with probability 0.90 and \$5 million with probability 0.10.

- If D is chosen, the agent receives \$0 with probability 0.89 and \$1 million with probability 0.11.

As Allais intuited, and as many researchers have experimentally confirmed[6], most people prefer B to A and C to D. Such preferences strike many as normatively unobjectionable. However, they violate Independence. To see this, let x_1 denote the outcome of receiving \$0, x_2 denote receiving \$1 million, and x_3 denote receiving \$5 million, and let $d^* = (\frac{1}{11}, x_1; \frac{10}{11}, x_3)$. Now observe that

- $d(A) = (0.89, x_2; 0.11, d^*)$,

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[3]Similarly, just as the Sure Thing Principle is analogous to Independence in Savage's framework, Grant et al. (2000) introduce Decomposability as an analogue of Betweenness in the Savage framework, and Buchak's (2010) Comonotonic Sure Thing Principle is analogus to Comonotonic Indpendence.

[4]This is not to say that Independence is the only controversial axiom of EU. See for example Tarsney (2020) for a proposed normative decision theory violating Completeness; Fishburn (1988) for a partial survey of decision theories violating Transitivity; or Arrhenius and Rabinowicz (2005) for a defense of violating Continuity.

[5]It was also originally motivated by rudimentary analogies between probability mixtures and mixtures of commodities. See, for example, Manne and Charnes (1952) and Wold (1952) for early formulations of such arguments against Independence. Yaari (1987, p. 95) rejects Independence, in part, because of the way that it results in an apparent conflation of attitudes towards risk and attitudes towards wealth.

[6]See for instance Morrison (1967), Raiffa (1968), and Slovic and Tversky (1974).

- $d(B) = (0.89, x_2; 0.11, x_2)$,

- $d(C) = (0.89, x_1; 0.11, d^*)$, and

- $d(D) = (0.89, x_1; 0.11, x_2)$.

Independence requires that an agent's preferences between d^* and x_2 not depend on whether she faces a background probability of x_2 or x_1. In other words, Independence requires $A \succ B \iff C \succ D$.

In the face of appealing but Independence-violating preferences, decision theorists must weigh the normative appeal of the Independence axiom against that of the preferences in question. A survey of the large literature on Independence lies outside the scope of this paper, but we will now outline the most relevant territory.[7]

3.3   Arguments for Independence

3.3.1   The "sure thing" argument

The earliest and simplest argument for Independence might be called the *sure thing argument*. The argument consists of an emphasis on the fact that Independence just requires an agent to prefer one act F to another G when, in the face of uncertainty about an arbitrary event, she would prefer F to G both if she knew that the event obtained and if she knew that it did not. That is, Independence just requires a preference of F to G when F is, in some sense, "surely" preferable to G.

The following illustration of this intuition is taken from Savage (1954, p. 21):

> A businessman contemplates buying a certain piece of property. He considers the outcome of the next presidential election relevant... [H]e asks whether he would buy if he knew that the Democratic candidate were going to win, and decides that he would do so. Similarly, he considers whether he would buy if he knew that the Republican candidate were going to win, and again finds that he would do so. Seeing that he would buy in either event... he should buy.

When presented this way, Savage then writes that "except possibly for the assumption of simple ordering, I know of no other extralogical principle governing decisions that finds such ready acceptance". Friedman and Savage (1952, p. 469) likewise guess that, after reflecting on the sure-thing reasoning above, a person will find that the Independence axiom "is not one he would deliberately violate".[8]

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[7]A thorough history of the early debate over Independence is given by Fishburn and Wakker (1995). A more recent overview and extension of some of the relevant arguments can be found in Buchak (2013).

[8]Though see Grant et al. (2000) for a contrary view.

At least a brief discussion of the sure thing argument for Independence can be found in almost any introduction to decision theory.

3.3.2 The "value of information" argument

Another argument for Independence leverages intuitions about how a rational agent responds to information. In particular, Independence follows from the plausible principle that "the acceptance of costless perfect information is a fundamental property of rational behaviour in a single person game against nature setting; to reject costless information in such a setting seems self-evidently irrational" (Keasey 1984, p. 648).

More formally: consider $n + 1$ k-sets of acts, denoted

$$\begin{aligned}
\Gamma &= \{F_1, ..., F_k\}, \\
\Gamma_1 &= \{F_{1,1}, ..., F_{1,k}\}, ..., \\
\Gamma_n &= \{F_{n,1}, ..., F_{n,k}\},
\end{aligned} \tag{3}$$

and a measurable n-partition $\mathscr{P}$ on S, where

$$d(F_i | \mathscr{P}_j) = d(F_{i,j}) \; \forall i \in I_k, \; \forall j \in I_n. \tag{4}$$

Now consider an act G such that

$$d(G) = (\mu(\mathscr{P}_1), d(B_1); ...; \mu(\mathscr{P}_n), d(B_n)), \tag{5}$$

where $B_j \in b(\Gamma_j) \; \forall j \in I_n$. Observe that G can be interpreted as the act of acquiring information, as given by $\mathscr{P}$, before choosing from Γ. The value of information argument maintains that rationality here *prima facie* requires $G \succsim F \; \forall F \in \Gamma$.[9]

Blackwell (1953) proves that an expected utility maximizer will, as desired, always accept costless information in a standard single-person decision. Good (1967) independently proves the same. (Though note that Good does not take the principle

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[9]Outside this "game against nature" setting, it is easy to find situations in which one seemingly does best to reject costless information. Most straightforwardly, one should reject information when accepting it would prompt an adverse response from others (e.g., if consulting a genetic health test may prompt one's health insurer may raise one's premium). But perhaps these adverse responses should be understood as "costs" to acquiring the information in question.

More subtly, consider "symptomatic" acts—those whose performance *predictably* allows the agent to rule out certain states. Such acts are impossible within the framework used here, which implicitly assumes that the chosen act is independent of the state. But they are undoubtedly possible in reality, and the two most common accounts of how to evaluate them both sometimes recommend information avoidance (as shown by Adams and Rosenkrantz (1980) and Maher (1990), respectively. Buchak (2013, p. 188) suggests that this weakens the "value of information" argument for Independence in general.

In any event, the case for accepting costless information is plausible enough in the restricted kind of case discussed in the main text, whether or not there is some more general requirement to accept costless information.

that one should accept costless information for granted; rather, his approach is to ground that principle in the more primitive principle that one should always maximize expected utility.) Wakker (1988) proves the converse: that if an agent violates Independence, she will necessarily sometimes be information-avoidant. Wakker takes this to be strong evidence in Independence's favor.

Further testimony to the normative force of the "value of information" argument for Independence is given by Kadane et al. (2008), Al-Najjar and Weinstein (2009), and Bradley and Steele (2016), who explore the implications of weakening Independence to allow for "ambiguity aversion"; by Hilton (1990), who explores the implications of Machina's (1982) proposal to replace Independence with a weaker "preference-smoothness" condition; and by Briggs (2016), who explores the implications of rank-dependent utility. In each case, the authors find information-aversion to be a puzzling, and possibly unacceptable, consequence of the respective weakening of Independence.

3.3.3   Other arguments

The case for Independence is made on many grounds beyond the two outlined above. Foremost among these are arguments to the effect that violations of Independence produce undesirable behavior in the context of sequential choice problems. For example, it is sometimes claimed that violations of Independence require an agent to make sequences of decisions which, viewed collectively, produce distributions of outcomes which the agent disprefers to other distributions of outcomes which were available to the agent all along.

We will therefore close this section by noting the discussion of these "sequential choice" arguments in Buchak (2013, ch. 6). Many think that rational agents evaluate each act in a sequential choice situation in light of the future anticipated choices, and that this avoids the alleged undesirable behavior caused by violations of Independence. Others think that rational agents evaluate each act in light of the past structure of a sequential choice situation, and that this avoids the alleged undesirable behavior caused by violations of Independence. We will not enter into that discussion (again, see Buchak (2013, ch. 6) for a helpful discussion), but focus on the relatively underexplored issue of how (various kinds of) Independence violations interact with purported constraints on the value of information.

4   Comonotonic Independence and RDU

4.1   Arguments for weakening Independence to Comonotonic Independence

We have sketched two primary arguments in favor of the Independence axiom: "sure thing" and "value of information". Some critics of Independence have argued that

neither argument holds in full generality. In particular, they claim that we should only accept weaker "comonotonic sure thing" and "value of comonotonic information" arguments, which support Comonotonic Independence.

4.1.1 Weakening the "sure thing" argument

The "sure thing" argument holds that whenever the outcome-distribution induced by one act G is weakly preferred to that induced by another act F both if some event $\mathcal{E}$ obtains and if it does not, then we should find $G \succsim F$ in general.

As McClennen (1990, ch. 3.8) and Buchak (2013, ch. 5.4) have pointed out, however, the argument is typically motivated by illustrations in which we are led to believe that not merely the *sub-distribution* but in fact the *outcome* resulting from F is guaranteed to be preferred to that resulting from G. In McClennen's words, "when various writers have sought to 'motivate' or rationalize the independence principle, they have typically illustrated it with reference to the special case in which components are sure (riskless) outcomes" (p. 59). Consider Savage's businessman again, for example: we might have imagined that he knows (or thinks he knows) roughly what the property will be worth in the event of each candidate's victory.

In such a case, perhaps, an intuitively undeniable "sure thing" argument would hold. This would amount to the criterion that G be preferred to F if G *statewise dominates* F. When G does not statewise dominate F, however—when there are states in which the outcome of F is preferred—the strength of the "sure thing" intuition is less clear. One might prefer F in the face of uncertainty about $\mathcal{E}$; maintaining one's choice to perform F upon learning whether $\mathcal{E}$ is simply *not sure* to result in an outcome preferred to that which will obtain if one performs G. One might therefore question whether such cases deserve the "sure thing" label at all.

Weakening Independence all the way to the criterion of Statewise Dominance, however, would classify an undesirably wide range of preferences as rationally permissible. Instead, therefore, Buchak (with many others uncomfortable with a wholesale endorsement of the "sure thing" argument for Independence) endorses an intermediate argument, which might be called the "comonotonic sure thing argument". This argument holds that, if acts F and G "agree on which states lead to better outcomes", and if an agent would prefer G conditional both on some event obtaining and on its failing to obtain, *then* the agent is rationally required to prefer G. The argument justifies a principle introduced by Schmeidler (1989), termed Comonotonic Independence.[10]

Definition 12. *A set of acts Δ is a <u>comoncone</u> if $u(x(F, s)) \succsim u(x(F, s')) \iff u(x(G, s)) \succsim u(x(G, s')) \; \forall F, G \in \Delta \; \forall s, s' \in S.$*

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[10]Hong and Wakker (1996) introduce the analogous Comonotonic Sure Thing Principle, constructed for the framework without objective probabilities. Yaari (1987, p. 104) gives a slightly different motivation for Comonotonic Independence.

That is, Δ is a comoncone if the acts in Δ order the states the same way, in terms of the utilities of the states' respective outcomes.

Definition 13. *An agent's preferences over a set of acts Γ satisfy the Comonotonic Independence axiom iff, for any five acts $F, G, H, K, L \in \Delta \subseteq \Gamma$ where $G \succsim F$, $d(K) = (p, d(F); 1 - p, d(H))$, $d(L) = (p, d(G); 1 - p, d(H))$, and Δ is a comoncone, $L \succsim K$.*

One might buy Buchak's criticism of the sure thing argument for Independence, but not her comonotonic sure thing argument for Comonotonic Independence. After all, one might think it unclear that Allais-like preferences are *irrational* even when all acts lie in the same comoncone. And as noted, if we are to weaken Independence, Comonotonic Independence is inconsistent with Betweenness. We state that axiom formally shortly, but in essence it says that for two acts F and G, if you are indifferent between them, then you are indifferent between each and a coin toss to decide between them. This may strike one as just as compelling as Comonotonic Independence, which would put pressure on the latter. So, just as one might be skeptical that Independence is a universal constraint, one might be skeptical that Comonotonic Independence is a universal constraint.

4.1.2 Weakening the "value of information" argument

Perhaps the "value of information" argument provides better guidance on how to go about weakening Independence. That argument holds that whenever an agent has the opportunity to gain costless information about the state of the world before deciding between F and G, he is rationally required to do so. Indeed, Buchak has provided an argument that we should accept only costless information *that is not potentially misleading*, and she takes this to support Comonotonic Independence.

As Buchak (2010; 2013, pp. 195–200) has pointed out, the principle that we should accept costless information is typically motivated by considering cases in which free information is guaranteed to lead us toward the act with the preferred outcome—or at least, guaranteed not to lead us away from it. In reality, however, information is often *misleading*, in the sense that it leaves us worse off, once our decision has been made, than we would have been without it. Consider, for example, the case given by Buchak (2010, pp. 85, 97):

> You are a shipowner. One day you are standing on the dock by your vessel, admiring the raging sea, when you notice that a small craft carrying nine people has capsized. Your ship can carry them all to safety, and if you do not rescue them, they will surely die. If you attempt to rescue them and your ship is not seaworthy, you will die along with them, but happily, you are almost certain that it is seaworthy. And even more happily, you have just enough time to perform a small test... testing for

rot on a part of the ship that is especially prone to rot but has little to do with the structural integrity of the ship.

When we imagine scenarios like these, designed to highlight information's potential to mislead, the strength of the "value of information" intuition is less clear.

The criterion that one accept information whenever it runs no risk of being misleading would naturally be formalized as the criterion that, if $F \succsim G$, one accept information given by partition $\mathscr{P}$ when the outcome-distribution induced by G conditional on some partition-element $\mathscr{P}_i$ is preferred to that induced by F conditional on $\mathscr{P}_i$ only if $x(G, s) \succsim x(F, s) \ \forall s \in \mathscr{P}_i$.

As with restricting "sure thing" reasoning to statewise dominance, this move would classify an undesirably wide range of preferences as rationally permissible. And again, Buchak endorses an intermediate argument, which might be called the "value of comonotonic information" argument. This holds that, even if information-avoidance is not irrational *in general*, it is irrational "when the possible information to be learned is partitioned into news that is of a similar nature" for all the acts under consideration (2013, p. 98). That is, information is always desirable when it is guaranteed not to be ambiguous across acts—when it is guaranteed either to be good news or to be bad news, with respect to the outcome that would result from any act. That way, if it is misleading about one act, it is misleading "in the same direction" about the others. It will, in some sense, not be misleading "about *the choice between*" acts.

It will be evident on reflection that this argument precisely justifies Comonotonic Independence.

4.2 Rank-dependent utility theory

Wakker et al. (1994) show that weakening Independence to Comonotonic Independence, and keeping the other axioms in place, is equivalent to Yaari's (1987) formalization of "rank-dependent utility".

Define a risk function $r : [0, 1] \rightarrow [0, 1]$ as a non-decreasing function over probabilities with $r(0) = 0$ and $r(1) = 1$. Then, given utility function $u(\cdot)$ and risk function $r(\cdot)$, define and denote the rank-dependent utility of an act F by

$$\mathrm{RDU}_{u,r}(F) \triangleq u\big(F(S)_1\big) + \sum_{i=2}^{|F(S)|} r\Big(\mu\big(F^{-1}(F(S)_i)\big)\Big)\Big(u\big(F(S)_i\big) - u\big(F(S)_{i-1}\big)\Big). \quad (6)$$

Proposition 3 (A representation theorem for RDU). *An agent's preferences over Φ satisfy Completeness, Transitivity, Continuity, and Comonotonic Independence iff they can be represented by a utility function over outcomes $u : X \rightarrow \mathbb{R}$, unique up to positive affine transformation, and a risk function $r(\cdot)$, such that $F \succsim G \iff RDU_{u,r}(F) \geq RDU_{u,r}(G)$.*[11]

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[11]Proof of this representation in the infinite-outcome case is given by Abdellaoui (2002).

Definition 14. *Rank-dependent utility theory is the claim that rational agents, for some utility function $u(\cdot)$ and some risk function $r(\cdot)$, maximize rank-dependent utility.*

Some appealing features of rank-dependent utility, such as the fact that it forbids the preference of stochastically dominated outcome-distributions, are identified by Quiggin (1982, 1993) and Yaari (1987) in originally introducing the theory. Wakker (1990) and Nakamura (1995) later find that RDU is roughly equivalent to "Choquet expected utility" (CEU), a decision theory developed to represent "ambiguity aversion" in the face of imprecise credences. Further discussion of the appealing features of rank-dependent utility lies outside the scope of this paper; Buchak (2013) offers a thorough exploration.

4.3   Risk-avoidance

Definition 15. *An RDU-maximizing agent is* anywhere-risk-avoidant *if, given his risk function $r(\cdot)$, $r(p) < p$ for some $p \in (0, 1)$.*

Risk-avoidance can rationalize the Allais preferences. In particular, observe that an RDU-maximizer exhibits the Allais preferences iff

$$\frac{r(0.1)}{1 + r(0.1) - r(0.99)} < \frac{u(x_2) - u(x_1)}{u(x_3) - u(x_1)} < \frac{r(0.1)}{r(0.11)} \tag{7}$$

(where, as above, x_1 denotes the outcome of receiving \$0, x_2 denotes the outcome of receiving \$1 million, and x_3 denotes the outcome of receiving \$5 million). These inequalities are satisfied, for example, when $r(p) = p^2$, $u(x_1) = 0$, $u(x_2) = 1$, and $u(x_3) = 2$.[12]

Segal (1987) identifies convexity in the risk function as a necessary ingredient for Allais-like preferences more generally.[13] He finds that convexity is also necessary to explain preferences that exhibit the "common ratio effect"—another commonly observed deviation from expected utility maximization.[14]

Finally, the "cumulative prospect theory" (CPT) of Tversky and Kahneman (1992) is by far the most widely accepted formalization of real-world human behavior in response to risk. CPT, once fitted to the experimental data, is equivalent to RDU with an "inverse-S-shaped" risk function and a privileged status-quo point,

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[12]This example is given by Buchak (2013, p. 71).

[13]He refers to "concavity in the weighting function" rather than "convexity in the risk function", because he uses the formalization of RDU given by Yaari (1987) (and others), whereas we are using that given by Buchak (2013) (and others). The two conditions are equivalent, however: this follows immediately from Buchak (2013, p. 57).

[14]Diecidue et al. (2009) establish that just the power weighting functions (i.e., functions of the form $r(p) = p^n$ for $n > 0$) exhibit Common Ratio Invariance, a constraint that rules out the common ratio effect.

where $r(p) = p$ at the point where an act under consideration begins to offer some chance of "gains" rather than merely greater or lesser losses (Buchak, 2013; pp. 59, 66). That is, people are observed to exhibit risk-inclination with respect to possible losses, but risk-avoidance with respect to possible gains.

In sum, the ability to accommodate at least some form of anywhere-risk-avoidance is widely understood to be a necessary feature of any well-motivated generalization of expected utility theory.

5 Betweenness

As discussed in §1, the other commonly proposed weakening of Independence is an axiom termed Betweenness.

Definition 16. *An agent's preferences over a set of acts Γ satisfy the <u>Betweenness</u> axiom iff, for any three acts $F, G, H \in \Gamma$ where $H \succsim F$ and $d(G) = (p, d(H); 1 - p, d(F))$ for some $p \in [0, 1]$, $H \succsim G \succsim F$.*

Betweenness is a simple axiom which is weak enough to permit risk-avoidance (as reflected e.g. by the Allais preferences) but strong enough, in conjunction with the other von Neumann-Morgenstern axioms, to forbid a wide range of seemingly irrational behavior (see Chew (1983, 1989)).[15] Its normative status comes from the plausible thought that randomization by itself is not instrumentally valuable or disvaluable—tossing a coin to decide between acts is never, say, better than performing either of those acts outright.[16]

Note that other weakenings of Independence generally either fail to permit the Allais preferences (see e.g. the Homotheticity axiom of Burghart et al. (2014)) or go so far as to permit more objectionable preferences, such as preferences for stochastically dominated utility-distributions (see e.g. Karmarkar's (1978, 1979) theory of "subjectively weighted utility"). This justifies our focusing the discussion on Betweenness and Comonotonic Independence.

6 Weak Betweenness

Definition 17. *An agent's preferences over a set of acts Γ satisfy <u>Weak Betweenness</u> iff, for any pair of acts and single simple act $F, G, h \in \Gamma$ where $h \succ F$ and $d(G) = (p, x(h); 1 - p, d(F))$ for some $p \in [0, 1]$, $G \succsim F$.*

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[15]Strictly speaking, Betweenness in conjunction with the other von Neumann-Morgenstern axioms is consistent with violations of first-order stochastic dominance. Nonetheless, Betweenness *permits* that we respect first-order stochastic dominance, while permitting risk-avoidance.

[16]See Bottomley and Williamson (forthcoming) and Gul and Lantto (1990) for further normative considerations in favor of Betweenness.

Note that, as stated here, Weak Betweenness is simply Betweenness with the requirement that H be simple. Betweenness-satisfying preferences thus satisfy Weak Betwenness. As we will see below, however, the converse does not hold; Weak Betweenness is indeed strictly weaker than Betweenness.

Proposition 4. *In the context of any decision theory that forbids the preference of stochastically dominated outcome-distributions, Weak Betweenness is equivalent to the following more general condition: For any pair of acts and set of simple acts $F, G, h_1, ..., h_n$ where $h_i \succ F \ \forall i \in I_n$ and $d(G) = (p_1, x(h_1); ...; p_n, x(h_n); 1 - \sum_{i=1}^{n} p_i, d(F))$ for some $p_1, ..., p_n \in [0, 1]$, $G \succsim F$.*

Proof: The backward implication is trivial. The forward implication follows from the fact that Weak Betweenness implies that $\underline{G} \succsim F$ when

$$d(\underline{G}) = \Big(\sum_{i=1}^{n} p_i, \underline{h}; 1 - \sum_{i=1}^{n} p_i, d(F) \Big), \tag{8}$$

where $\underline{h}$ is a least-preferred simple act among $\{h_1, ..., h_n\}$, and the fact that G stochastically dominates $\underline{G}$. ∎

We can interpret Weak Betweenness as the condition that, if one has the costless opportunity to receive certainty about the outcome of some act H (without learning anything about the outcome of F) before deciding between H and F, one should not prefer to reject this opportunity.

6.1 Arguments for Weak Betweenness

On both lines of reasoning explored above, Weak Betweenness stands out as a natural weakening of Independence which more naturally addresses the objections raised by defenders of Comonotonic Independence than does Comonotonic Independence itself.

6.1.1 The "sure thing" argument

The "sure thing" motivation for Weak Betweenness is simple. If there is an event $\mathcal{E}$ such that the outcome of G is preferred to the outcome distribution of F if $\mathcal{E}$ obtains, and such that the outcome-distribution of G is identical to that of F if $\mathcal{E}$ does not obtain, it seems natural to say that G is surely preferable to F. More precisely, it seems like a natural step between mere statewise dominance and the "sub-act dominance" condition codified as Independence. G improves on F, the argument goes, by assigning each state in $\mathcal{E}$ to a known outcome preferred to act F as a whole.

Comonotonic Independence, by contrast, requires rational preferences over acts to obey full "sub-act dominance"—but only when all the acts in question are comonotonic. As several authors have noted (e.g. Luce, 1996a, 1996b; Safra and Segal, 1987),

it is difficult to find a natural explanation of why this particular restriction would be desirable. Some (e.g. Diecidue and Wakker, 2001) attempt one; but we expect most readers, including those most sympathetic to Comonotonic Independence, will not find these as natural as the justification for Weak Betweenness.

6.1.2 The "value of information" argument

Recall the argument that it may be rational to avoid information out of fear that the information will be misleading—that it may lead us to consider a lottery more or less valuable than it in fact is. Even if we accept this argument, we may still be inclined to accept Weak Betweenness, since it only requires a rational agent to accept information that will pin down the value of one act with certainty and shed no light on any other act. Turning down information may sometimes strike us as appealing, but doing so even when the information comes with *no possibility of misleading* us about any of the acts available to us appears especially troublesome.

Comonotonic Independence, by contrast, requires us to accept information whenever it is "of a similar nature" for the acts under consideration (i.e. "good news" or "bad news"). There is simply no straightforward sense in which this restricts the potential of the news to be misleading. As the reader can easily verify, given any utility function $u(\cdot)$ over outcomes X and any risk function $r(\cdot)$, for any act F that does not guarantee a best outcome there is a comonotonic act H, an act G, an information partition $\mathscr{P}$, and a state s such that $d(G) = d(H|\mathscr{P}(s))$, $\mathrm{RDU}_{u,r}(G) > \mathrm{RDU}_{u,r}(F)$, and $u(H(s)) = u(G(s)) \leq u(x) \ \forall x \in X$. That is, except in the special case that F guarantees a best outcome, accepting comonotonic information can always mislead an agent into choosing an act with a *worst* outcome. By contrast, accepting perfect information about H can never result in an outcome dispreferred to F, and therefore it can never result in an outcome worse than the worst outcome possible under F.

Finally, one might point out that, even if Weak Betweenness is more promising than Comonotonic Independence as a rational requirement in response to information, Weak Betweenness still (like Comonotonic Independence) exposes an agent to the risk of switching from F to an act with a worse outcome (in the event that $v(F) > v(H)$). One might argue that Weak Betweenness too, therefore, fails in some sense to protect the agent from misleading information. In response, rather than parse the word "misleading", recall the observation that the "sure thing" argument for Independence is typically illustrated by cases in which not merely the outcome-distribution, but the outcome, of F is surely preferred to G regardless of whether an event $\mathcal{E}$ obtains. The implication, in this instance, is that the case for accepting Independence over arbitrary sub-acts is weaker than we had been led to believe. Likewise, then, observe that the argument for rejecting misleading information is typically illustrated (Buchak, 2010; 2013, p. 193) by cases in which one has the opportunity to gain some information about the outcome-distribution for an act, but not to fully identify its outcome.

Partially examining the rot on one's ship runs the risk that one will let the strangers needlessly drown; *fully* determining the ship's seaworthiness, and thus the outcome of attempting a rescue, seems like a much stranger opportunity to dismiss. This suggests that the case for rejecting *perfect* information about acts is *weaker* than we may have been led to believe. In other words, Weak Betweenness does not ask us to do what troubles us when we seek to avoid misleading information.

A more thorough debate along these lines may be possible. For now we will simply trust that the force of the intuition for Weak Betweenness is clear, and hope that it is felt by at least a few who are sympathetic to Comonotonic Independence but not generally persuaded of an obligation to maximize expected utility.

7   Results

We can now state the following:

Proposition 5. *An RDU-maximizer violates Weak Betweenness if she is anywhere-risk-avoidant and her risk function is either continuous or strictly increasing.*

Proof: Consider an anywhere-risk-avoidant RDU-maximizer with utility function $u(\cdot)$ and risk function $r(\cdot)$. Let

$$\tilde{r}(p) \triangleq r(p) - p, \quad p \in [0, 1]. \tag{9}$$

We know that $\tilde{r}(p) < 0$ for some $p \in (0, 1)$, by definition of anywhere-risk-avoidance, and by the fact that $\tilde{r}(0) = \tilde{r}(1) = 0$. Given such a p, furthermore, because $r(\cdot)$ is weakly increasing, we have $\tilde{r}(\tilde{p}) < 0 \ \forall \tilde{p} \in (r(p), p)$. Also because $r(\cdot)$ is weakly increasing, it is continuous (and indeed differentiable) almost everywhere, by Lebesgue's theorem for the differentiability of monotone functions. So therefore is $\tilde{r}(\cdot)$. There thus exists a $p \in (0, 1)$ with $\tilde{r}(p) < 0$ around which $\tilde{r}(\cdot)$ is locally continuous. Denote some such p by p_0.

Because $r(p_0) < 0$, $r(1) = 0$, and $r(\cdot)$ is locally continuous around p_0, there exists a $p^* \in (p_0, 1)$ with

$$r(\cdot) \text{ locally continuous throughout } [p_0, p^*], \tag{10}$$

$$\tilde{r}(p) < 0 \quad \forall p \in [p_0, p^*], \tag{11}$$

$$\text{and} \quad \tilde{r}(p^*) \in (\tilde{r}(p_0), 0). \tag{12}$$

Then by continuity of $\tilde{r}(\cdot)$ around p^*, there is an open interval $P^* \ni p^*$ such that (12) and (11) hold for all $p \in P^*$ in place of p^*.

Choose any such P^*, and then choose any $\bar{p} \in P^*$. Define

$$P \triangleq [p_0, \bar{p}], \tag{13}$$

$$a(p) \triangleq \frac{1 - r(p)}{1 - p}, \quad p \in P. \tag{14}$$

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Because $a(\cdot)$ is continuous on P and P is compact,

$$\underline{a} \triangleq \min_{p \in P} a(p) \tag{15}$$

is defined. Also, observe that for all $p \in P$, we have $r(p) < p$, and thus $a(p) > 1$. So $\underline{a} > 1$. We will now show that there exists a $p \in P$ and a $q \in (0, 1)$ such that

$$r(p)r(p + (1 - p)q) + (1 - r(p))r((1 - q)p) < r(p). \tag{16}$$

Given such a $\bar{p}$ and the corresponding P, suppose by contradiction that there is no $p \in P$ and $q \in (0, 1)$ for which (16) holds. Also, given $p_n \in [p_0, \bar{p})$, define

$$p_{n+1} \triangleq \frac{p_n}{1 - \bar{p} + p_n}, \tag{17}$$

$$q_{n+1} \triangleq \bar{p} - p_n. \tag{18}$$

Observe that (17)–(18) imply

$$(1 - q_{n+1})p_{n+1} = p_n, \tag{19}$$

$$p_{n+1} + (1 - p_{n+1})q_{n+1} = \bar{p}, \tag{20}$$

$$p_{n+1} \in (p_n, \bar{p}) \subset P, \tag{21}$$

$$q_{n+1} \in (0, 1). \tag{22}$$

By (12), $r(\bar{p}) > r(p_0)$, so

$$\frac{r(\bar{p}) - r(p_0)}{\bar{p} - p_0} > 0. \tag{23}$$

Also, by our contradictory supposition and (19)–(22), for all $n \geq 0$ we have

$$r(p_{n+1})r(\bar{p}) + (1 - r(p_{n+1}))r(p_n) \geq r(p_{n+1}) \tag{24}$$

$$\implies r(\bar{p})(1 - r(p_{n+1})) - r(p_n)(1 - r(p_{n+1})) \leq r(\bar{p}) - r(p_{n+1}) \tag{25}$$

$$\implies \frac{r(\bar{p}) - r(p_{n+1})}{\bar{p} - p_{n+1}} \geq \frac{r(\bar{p}) - r(p_n)}{\bar{p} - p_n}(1 - r(p_{n+1}))\frac{\bar{p} - p_n}{\bar{p} - p_{n+1}}. \tag{26}$$

By (17),

$$\frac{\bar{p} - p_n}{\bar{p} - p_{n+1}} = \frac{1}{1 - p_{n+1}}. \tag{27}$$

Also, since $p_{n+1} \in P$,

$$\frac{1 - r(p_{n+1})}{1 - p_{n+1}} = a(p_{n+1}) \geq \underline{a} > 1. \tag{28}$$

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It then follows from (23) and (26)–(28) that

$$\lim_{n \to \infty} \frac{r(\bar{p}) - r(p_n)}{\bar{p} - p_n} = \infty. \tag{29}$$

By (17),

$$p_{n+1}\bar{p} + (1 - p_{n+1})p_n = p_{n+1}; \tag{30}$$

i.e. p_{n+1} always covers fraction p_{n+1} of the distance from p_n up to $\bar{p}$. By (21), this fraction is bounded above 0. So the sequence $\{p_n\} \to \bar{p}$. It follows that $r(\cdot)$ is not differentiable at $\bar{p}$.

The above holds for all $\bar{p} \in P^*$, which is of positive measure. But because $r(\cdot)$ is weakly increasing, it must be differentiable almost everywhere, by Lebesgue's theorem for the differentiability of monotone functions. So there exists a $p \in P$ and a $q \in (0, 1)$ such that (16) holds.

Choose such a p and q, and define an act F such that

$$d(F) = (1 - p, x_1; p, x_2), \tag{31}$$

where, without loss of generality, $u(x_1) = 0$ and $u(x_2) = 1$. Choose $y \in (0, 1 - r(p))$ such that

$$\big(r(q+(1-q)p) - r((1-q)p)\big)y < r(p) - \big(r(p)r(q+(1-q)p) + (1-r(p))r((1-q)p)\big). \tag{32}$$

Note that the right-hand side is strictly positive, by (16) and our choices of p and q, and that the left-hand coefficient on y is weakly positive, since $(1 - q)p < p < q + (1 - q)p$ and $r(\cdot)$ is weakly increasing. Rearranging, we have

$$r(q + (1 - q)p)(r(p) + y) + r((1 - q)p)(1 - (r(p) + y)) < r(p). \tag{33}$$

Let $m \triangleq r(p) + y$, so that

$$r(q + (1 - q)p)m + r((1 - q)p)(1 - m) < r(p). \tag{34}$$

Because $y \in (0, 1 - r(p))$, $m \in (0, 1)$.

Let x be an outcome such that $u(x) = m$. Such an outcome must exist, by the assumption of continuity in outcomes, since $m \in [u(x_1), u(x_2)]$. Let h be a simple act such that $x(h) = x$. Also, define an act G such that $d(G) = (q, x; 1 - q, d(F))$. Observe that

$$\mathrm{RDU}_{u,r}(h) = m > r(p), \tag{35}$$
$$\mathrm{RDU}_{u,r}(F) = r(p), \tag{36}$$
$$\mathrm{RDU}_{u,r}(G) = r(q + (1 - q)p)m + r((1 - q)p)(1 - m). \tag{37}$$

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We have an act G, a simple act h, and a probability q such that $h \succ F$ (by (35), (36)), $d(G) = (q, x; 1 - q, F)$, and $G \prec F$ (by (34), (36), (37)). Our agent thus violates Weak Betweenness. ∎

Note that we have let F be an arbitrary two-outcome act, offering the preferred outcome with any probability p so long as (16) holds for some $q \in (0, 1)$; and, given this lottery, we have discovered that there is an opportunity for certainty which our RDU-maximizer would turn down. Furthermore, it is relatively straightforward to show that (16) holds, given sufficiently small q, for all p at which

- $r'(p)$ is defined and positive and

- $r(p) < p$.

Thus, we have done more than show that an anywhere-risk-avoidant RDU agent will *sometimes* be inclined to act on a violation of Weak Betweenness. We have also discovered something further about an "everywhere-risk-avoidant" RDU agent whose risk function always has a positive derivative within $(0, 1)$—e.g. whose risk function is

$$r(p) = p^b \text{ for some } b > 1, \tag{38}$$

to use the class of risk functions most commonly used in illustrations. In particular, we have discovered that *whenever* such an agent faces an action with two possible outcomes, there is a possible alternative that he would prefer to the action before him, but which he would avoid the chance to discover. Weak-Betweenness-violating behavior, therefore, is not restricted to curious, artificially constructed edge cases. There is a sense in which risk-avoidant agents reject opportunities for certainty pervasively.

Now we will show that Weak Betweenness permits a wide range of rank-dependent deviations from expected utility maximization. In doing so, we will demonstrate that Weak Betweenness is in fact substantively weaker than Betweenness, and not merely a new rhetorical justification for a principle of rational behavior which defenders of RDU theory have already knowingly rejected.

Proposition 6. *An RDU-maximizer with utility function $u(\cdot)$ and risk function $r(\cdot)$ obeys Weak Betweenness if $r(\cdot)$ is twice differentiable and everywhere concave.*

Proof: Let $r(\cdot)$ be twice differentiable and everywhere concave, and let F be an arbitrary act where

$$d(F) = (p_1, x(F)_1; ...; p_i, x(F)_i; ...; p_n, x(F)_n). \tag{39}$$

Let $x(F)_i$ denote an outcome with $u(x(F)_i) = \mathrm{RDU}_{u,r}(F)$. That is, if F offers any probability of this outcome, this probability is p_i. If not, $p_i = 0$. Now, given arbitrary $q \in [0, 1]$, let

$$d_F(q) \triangleq (q, x(F)_i; 1 - q, d(F)), \tag{40}$$

or, expanded,

$$d_F(q) = ((1-q)p_1, x(F)_1; ...; (1-q)p_i + q, x(F)_i; ...; (1-q)p_n, x(F)_n). \tag{41}$$

From this we have

$$
\begin{aligned}
\mathrm{RDU}_{u,r}(d_F(q)) &= u(x(F)_1) \\
&\quad + r\Big((1-q)\sum_{j=2}^{n} p_j + q\Big)\big(u(x(F)_2) - u(x(F)_1)\big) \\
&\quad + ... + r\Big((1-q)\sum_{j=i}^{n} p_j + q\Big)\big(u(x(F)_i) - u(x(F)_{i-1})\big) \\
&\quad + r\Big((1-q)\sum_{j=i+1}^{n} p_j\Big)\big(u(x(F)_{i+1}) - u(x(F)_i)\big) \\
&\quad + ... + r\big((1-q)p_n\big)\big(u(x(F)_n) - u(x(F)_{n-1})\big). \tag{42}
\end{aligned}
$$

Taking the second derivative with respect to q,

$$
\begin{aligned}
(\mathrm{RDU}_{u,r} \circ d_F)''(q) &= \Big(1 - \sum_{j=2}^{n} p_j\Big)^2 r''\Big((1-q)\sum_{j=2}^{n} p_j + q\Big)\big(u(x(F)_2) - u(x(F)_1)\big) \\
&\quad + ... + \Big(1 - \sum_{j=i}^{n} p_j\Big)^2 r''\Big((1-q)\sum_{j=i}^{n} p_j + q\Big)\big(u(x(F)_i) - u(x(F)_{i-1})\big) \\
&\quad + \Big(-\sum_{j=i+1}^{n} p_j\Big)^2 r''\Big((1-q)\sum_{j=i+1}^{n} p_j\Big)\big(u(x(F)_{i+1}) - u(x(F)_i)\big) \\
&\quad + ... + (-p_n)^2 \, r''\big((1-q)p_n\big)\big(u(x(F)_n) - u(x(F)_{n-1})\big). \tag{43}
\end{aligned}
$$

Since r'' is always nonpositive (by $r(\cdot)$'s concavity), the above expression is always nonpositive. That is, $\mathrm{RDU}_{u,r} \circ d_F$ is concave in q. And since $(\mathrm{RDU}_{u,r} \circ d_F)(0) = (\mathrm{RDU}_{u,r} \circ d_F)(1) = \mathrm{RDU}_{u,r}(F)$, we know that, for any act G such that $d(G) = (q, x(F)_i; 1 - q; d(F))$ for some $q \in (0, 1)$, $G \succsim F$.

Finally, since RDU forbids the preference of stochastically dominated outcome-distributions, it follows that, for any pair of acts and simple act F, G, h such that $h \succ F$ and $d(G) = (q, x(h); 1 - q; d(F))$ for some $q \in [0, 1]$, $G \succsim F$. RDU agents with concave, twice-differentiable risk functions thus obey Weak Betweenness. ∎

8   Conclusion

We have defined a new candidate axiom for normative decision theory, termed Weak Betweenness. We have demonstrated that it is substantively weaker than the Independence axiom to which critics of expected utility theory commonly object, and strictly weaker even than the Betweenness axiom to which Independence is often weakened. We have argued that this weaker axiom is particularly normatively compelling because it avoids one of the main objections leveled at Independence: it does not ask an agent to risk exposing herself to "misleading" information about any of the acts available to her, but only to be willing to accept perfect information about some act's value. Finally, we have proven that accepting this axiom, in the context of rank-dependent utility theory, forbids risk-avoidant behavior (under a mild differentiability condition on the risk function).

Where does this leave the proponent of RDU? Comonotonic Independence and Weak Betweenness are jointly inconsistent with the kinds of preferences that typically motivate rejections of expected utility theory in the first place. So, if the proponent of RDU is sufficiently convinced of the arguments for Comonotonic Independence, Weak Betweenness may be enough to rule out all plausible non-linear risk functions, and recommend a return to the fold of expected utility theory. On the other hand, an RDU-proponent might be motivated not by the strength of the case for Comonotonic Independence in particular but simply by the desire to accommodate reasonable-seeming, risk-avoidant preferences. In this case, the results here suggest exploring the normative potential of Betweenness-(or at least Weak-Betweenness-)satisfying theories outside a rank-dependent framework. Such explorations remain relatively neglected by normative theorists and may be fruitful avenues for future research.

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